Typical MC 🧩 Example 2A[JR] (Polynomials and Algebraic Fractions)

$$\frac{2}{4-3x} - \frac{3}{4+3x} =$$

   A. $\dfrac{-4}{16-9x^2}$.

   B. $\dfrac{-4-3x}{16-9x^2}$.

   C. $\dfrac{-4+6x}{16-9x^2}$.

   D. $\dfrac{-4+15x}{16-9x^2}$.

Ref. (DSE 2023 Q2)

Answer: D

📊 Analysis

- To perform the addition or subtraction of algebraic fractions, we need to convert the fractions into the same denominator first.

Solution:

$$\frac{2}{4-3x} - \frac{3}{4+3x} = \frac{2(4+3x)}{(4-3x)(4+3x)} - \frac{3(4-3x)}{(4-3x)(4+3x)}$$

$$= \frac{2(4+3x) - 3(4-3x)}{(4-3x)(4+3x)}$$

$$= \frac{2(4+3x) - 3(4-3x)}{16-9x^2} \qquad \blacktriangleleft (4-3x)(4+3x) = 4^2 - (3x)^2 = 16 - 9x^2$$

$$= \frac{2(4) + 2(3x) - 3(4) - 3(-3x)}{16-9x^2}$$

$$= \frac{8 + 6x - 12 + 9x}{16-9x^2}$$

$$= \frac{-4+15x}{16-9x^2}$$

✗ Common Mistake

- Wrongly expand $2(4+3x) - 3(4-3x)$ as $2(4) + 2(3x) - 3(4) - 3(3x)$.

✓ Correct Concept

- $2(4+3x) - 3(4-3x) = 2(4) + 2(3x) - 3(4) - 3(-3x)$
$$= 8 + 6x - 12 + 9x$$
$$= -4 + 15x$$

Typical MC **Example** **2B** ^{JR} **(Polynomials and Algebraic Fractions)**

e - Example

$(3x^2 + 2x - 1) - 3(x^2 - 2x + 1) =$

- A. 0.
- B. $8x - 4.$
- C. $-4x + 2.$
- D. $6x^2 - 4x - 4.$

Answer: B

Analysis

- When we perform the operations of polynomials, we should be aware of the order of the operations:
  - (i)   perform the operations inside the brackets first,
  - (ii)   then multiplication and division, from left to right,
  - (iii) then addition and subtraction, from left to right.
- To subtract polynomials, remember to change signs when removing brackets.

Solution:

$(3x^2 + 2x - 1) - 3(x^2 - 2x + 1)$

$= 3x^2 + 2x - 1 - 3x^2 + 6x - 3$ ◄ Remove the brackets.

$= 3x^2 - 3x^2 + 2x + 6x - 1 - 3$ ◄ Group the like terms together.

$= 8x - 4$

✗ Common Mistake

- Wrongly expand $-3(x^2 - 2x + 1)$ as $-3x^2 - 2x + 1$ or $-3x^2 - 6x + 3.$
- Wrongly simplify $3x^2 + 2x - 1 - 3x^2 + 6x - 3.$

✔ Correct Concept

- $-3(x^2 - 2x + 1) = -3x^2 - 3(-2)x - 3(1)$
  $\qquad\qquad\qquad = -3x^2 + 6x - 3$
- $3x^2 + 2x - 1 - 3x^2 + 6x - 3$
  $= 3x^2 - 3x^2 + 2x + 6x - 1 - 3$
  $= 8x - 4$

Exam Practice

Choose the **BEST** answer for each question.

1. Laws of Integral Indices

1. $\left(\dfrac{1}{8}\right)^{600}(2^{600})^4 =$

   A. 0.   B. 2^{600}.

   C. 4^{600}.   D. 2^{800}.

2. $\left(\dfrac{-1}{6}\right)^{2013}(6^{2014}) =$

   A. -6.   B. 6.

   C. $-\dfrac{1}{6}$.   D. 0.

3. $9^{333} \cdot 7^{666} =$

   A. 21^{666}.   B. 21^{999}.

   C. 63^{666}.   D. 63^{999}.

4. $(3n^2)^{-3} =$

   A. $\dfrac{1}{9n^8}$.   B. $\dfrac{1}{9n^9}$.

   C. $\dfrac{1}{27n}$.   D. $\dfrac{1}{27n^6}$.

5. $3a^2 \times (3a)^2 =$

   A. $9a^4$.   B. $27a^3$.

   C. $27a^4$.   D. $81a^3$.

6. $(2a^2)^4 \times a^5 =$

   A. $8a^{13}$.   B. $8a^{21}$.

   C. $16a^{13}$.   D. $16a^{21}$.

7. $\dfrac{5x^2}{(5x^4)^4} =$

   A. $\dfrac{1}{125x^{14}}$.   B. $\dfrac{1}{125x^8}$.

   C. $\dfrac{1}{4x^6}$.   D. $\dfrac{1}{4x^4}$.

8. $\dfrac{4k^{-5}}{(9k^2)^{-2}} =$

   A. $\dfrac{4}{81k^5}$.   B. $\dfrac{4}{81k}$.

   C. $\dfrac{324}{k^5}$.   D. $\dfrac{324}{k}$.

9. $\dfrac{4^{3n+5}}{8^{2n+3}} =$

   A. 1.   B. 2.

   C. 2^n.   D. 2^{-n}.

10. $(16 \cdot 8^{n-1})^3 =$

   A. 2^{9n+1}.   B. 2^{9n+3}.

   C. 2^{18n+3}.   D. 2^{18n+6}.

11. $5^n \cdot 3^{2n} =$

   A. 15^n.   B. 30^n.

   C. 45^n.   D. 15^{2n}.

12. $\dfrac{a^{n-2} - a^{n-1}}{a^{n-2}} =$

   A. $-a^{n-1}$.
   B. $1 - a^{n-1}$.
   C. $1 - a$.
   D. $1 - \dfrac{1}{a}$.

13. If $9^x = 27^y$, then $x : y =$

   A. $1 : 3$.
   B. $2 : 3$.
   C. $3 : 1$.
   D. $3 : 2$.

2.   Polynomials and Algebraic Fractions

14. $(a + a + a)(b + b + b + b) =$

   A. $3a + 4b$.
   B. $a^3 b^4$.
   C. $12ab$.
   D. $12a^3 b^4$.

15. $x^3(3x + x) =$

   A. $4x^4$.
   B. $3x^5$.
   C. $4x^5$.
   D. $2x^6$.

DSE PP Q1

16. $(4x^2 - 2x - 3) - 4(x^2 + 3x - 2) =$

   A. $-5x + 1$.
   B. $-14x + 5$.
   C. $8x^2 - 7x + 3$.
   D. $8x^2 - 14x + 5$.

17. $(x^2 - 3x + 6)(x + 2) =$

   A. $x^3 - x^2 + 12$.
   B. $x^3 + x^2 + 12$.
   C. $x^3 - x^2 - 12x + 12$.
   D. $x^3 + x^2 + 12x + 12$.

DSE 2015 Q1

18. $(2x - 3)(4x^2 + 3x - 2) =$

   A. $8x^3 + 18x^2 - 5x + 6$.
   B. $8x^3 + 6x^2 + 13x + 6$.
   C. $8x^3 - 18x^2 + 5x + 6$.
   D. $8x^3 - 6x^2 - 13x + 6$.

19. $(a + 3b)(a - 3b - 4) =$

   A. $a^2 - 3b^2 - 4a - 12b$.
   B. $a^2 - 9b^2 - 4a - 12b$.
   C. $a^2 - 9b^2 - 4a + 12b$.
   D. $a^2 - 9b^2 + 4a - 12b$.

20. $\dfrac{1}{a} - \dfrac{1}{a + 1} =$

   A. $\dfrac{1}{a(a + 1)}$.
   B. $-\dfrac{1}{a(a + 1)}$.
   C. $\dfrac{a}{a(a + 1)}$.
   D. $-\dfrac{a}{a(a + 1)}$.

21. $\dfrac{1}{4x - 3} - \dfrac{1}{4x + 3} =$

   A. $\dfrac{6}{4x^2 - 3}$.
   B. $\dfrac{8x}{4x^2 - 3}$.
   C. $\dfrac{6}{16x^2 - 9}$.
   D. $\dfrac{8x}{16x^2 - 9}$.

DSE 2018 Q4

22. $\dfrac{1}{x - 2} + \dfrac{1}{x + 2} =$

   A. $\dfrac{4}{x^2 - 2}$.
   B. $\dfrac{2x}{x^2 - 2}$.
   C. $\dfrac{4}{x^2 - 4}$.
   D. $\dfrac{2x}{x^2 - 4}$.

Selected Questions

56. $\dfrac{(5^n)(625^{2n})}{125^n} =$

    A. 25^{-n}.   B. 25^{-2n}.

    C. 25^{3n}.   D. 25^{6n}.

DSE 2021 Q1

57. $\dfrac{(256^{2m-1})^2}{64^{4m-3}} =$

    A. 4.   B. 4^{4m+1}.

    C. 4^{4m+5}.   D. 4^{8m+7}.

DSE 2022 Q2

58. $5^{2n+6} 9^{n+3} =$

    A. 15^{2n+6}.   B. 15^{4n+12}.

    C. 45^{2n+6}.   D. 45^{3n+9}.

DSE 2023 Q3

59. $(1-m)(m+n)(n-m) =$

    A. $m^3 - mn^2 + m^2 - n^2$.

    B. $m^3 - mn^2 - m^2 + n^2$.

    C. $-m^3 + mn^2 - m^2 + n^2$.

    D. $-m^3 + mn^2 + m^2 - n^2$.

DSE 2021 Q3

60. $(x+y)(x^2 - xy - y^2) =$

    A. $x^3 + y^3$.

    B. $x^3 - 2xy^2 - y^3$.

    C. $x^3 - 2x^2y + y^3$.

    D. $(x+y)^3$.

DSE 2019 Q1

61. $\dfrac{5}{a-5} - \dfrac{8}{a-8} =$

    A. $\dfrac{3a}{(a-5)(a-8)}$.   B. $\dfrac{3a}{(a-5)(8-a)}$.

    C. $\dfrac{3a+80}{(a-5)(a-8)}$.   D. $\dfrac{3a+80}{(a-5)(8-a)}$.

DSE 2021 Q4

62. $\dfrac{2}{6-7a} - \dfrac{3}{6+7a} =$

    A. $\dfrac{-6+7a}{36-49a^2}$.   B. $\dfrac{-6-7a}{36-49a^2}$.

    C. $\dfrac{-6+35a}{36-49a^2}$.   D. $\dfrac{-6-35a}{36-49a^2}$.

DSE 2023 Q2

63. If $x(y-1) = y(x-y)$, then $x =$

    A. y.   B. $2y$.

    C. y^2.   D. $\dfrac{y^2}{1-2y}$.

DSE 2021 Q2

64. If $p = 1 - \dfrac{1}{1+a}$, then $a =$

    A. $\dfrac{1}{1-p}$.   B. $\dfrac{1}{1+p}$.

    C. $\dfrac{p}{1-p}$.   D. $\dfrac{p}{1+p}$.

DSE 2019 Q5

65. If $\dfrac{2x+5y}{2x+7y} = \dfrac{3}{4y+1}$, then $x =$

    A. $\dfrac{5y^2-4y}{1+2y}$.   B. $\dfrac{5y^2-4y}{1-2y}$.

    C. $\dfrac{1+2y}{5y^2-4y}$.   D. $\dfrac{1-2y}{5y^2-4y}$.

DSE 2023 Q1

66. If c and d are constants such that
    $(x - 3)(x + c) + 8 \equiv x(x + 3d) - 2c(x + 1)$,
    then $d =$

    A. 7.   B. 8.
    C. –8.   D. –9.

    DSE 2023 Q5

67. If a and b are constants such that
    $(2x - 3)^2 + ax \equiv (4x - b)(x - 2) + b$,
    then $a =$

    A. 1.   B. 3.
    C. 17.   D. 23.

    DSE 2022 Q3

68. If a, b and c are non-zero constants such
    that $a(5x + 1) + b(x + 5) \equiv c(x + 3)$,
    then $a : b =$

    A. 1 : 5.   B. 1 : 7.
    C. 5 : 1.   D. 7 : 1.

    DSE 2021 Q6

69. $(6x - 7y)(8x - y) + x(18x - 21y) =$

    A. $(6x + 7y)(11x + y)$.
    B. $(6x + 7y)(5x + y)$.
    C. $(6x - 7y)(11x - y)$.
    D. $(6x - 7y)(5x - y)$.

    DSE 2020 Q4

70. $x^2 - 3xy + 2y^2 - 3x + 3y =$

    A. $(x + y)(x - 2y - 3)$.
    B. $(x + y)(x + 2y + 3)$.
    C. $(x - y)(x - 2y - 3)$.
    D. $(x - y)(x + 2y + 3)$.

    DSE 2023 Q4