Comparison between NEW and OLD syllabuses

The NEW Additional Mathematics syllabus is extracted from the old one. No new topics are added, but some topics are cut or trimmed. All the contents about 'Complex Numbers' as well as 'Conic Sections', such as ellipse, parabola and hyperbola had been removed from the syllabus. The changes of the topics of the new syllabus are listed in the following table:

Chapters	Topics of the syllabus	Topics removed
1. Quadratic Equations, Quadratic Functions and Absolute Values	• Quadratic functions and quadratic equations • Discriminant and nature of roots • Use of the absolute value sign	— — • Use of absolute value sign in relation to inequalities is not required.
2. Inequalities	• Quadratic inequalities in one variable	• Inequalities of the form $\frac{ax+b}{cx+d} \geq k$ are not required
3. Mathematical Induction	• Mathematical induction and its simple applications	—
4. Binomial Theorem	• The binomial theorem for positive integral indices	—
5. Trigonometry	• The six trigonometric functions of angles of any magnitude and their graphs • Formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$, sum and product formulae • General solution of simple trigonometric equations	— • Students are not required to prove these formulae. Their applications to multiple and half angles are expected but students are not required to memorize 'triple angle formulae' and 'half angle formulae' —
6. Solution of Triangles and its Applications	• Trigonometric problems in two- and three-dimensions	—

1 Quadratic Equations, Quadratic Functions and Absolute Values

Concept Map

Quadratic Equations, Quadratic

Quadratic equations

Methods of solving quadratic equations

- Factorization

  If $(mx+n)(px+q)=0$, then $x=-\frac{n}{m}$ or $-\frac{q}{p}$.
- Quadratic formula

  The roots of the quadratic equation $ax^2+bx+c=0\ (a\neq 0)$ are $x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$.

Nature of the roots of quadratic equations

Discriminant $\Delta=b^2-4ac$ determines the nature of roots of the quadratic equation $ax^2+bx+c=0 \cdots\cdots (*)\ (a\neq 0)$

- $\Delta>0$, $(*)$ has 2 unequal real roots
- $\Delta=0$, $(*)$ has 2 equal real roots
- $\Delta<0$, $(*)$ has no real roots

Sum and product of roots

- Let α and β be the roots of $ax^2+bx+c=0\ (a\neq 0)$, we have

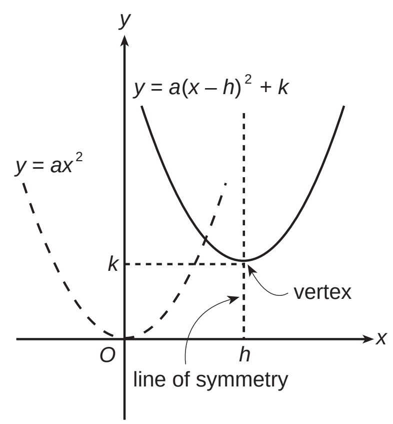
  $\alpha+\beta=-\frac{b}{a}$ and $\alpha\beta=\frac{c}{a}$

Method of the formation of quadratic equations

- If the roots of a quadratic equation is given, then the quadratic equation is $x^2-(\text{sum of roots})\,x+(\text{product of roots})=0$

Graphs of quadratic functions

Shape of $y=a(x-h)^2+k$

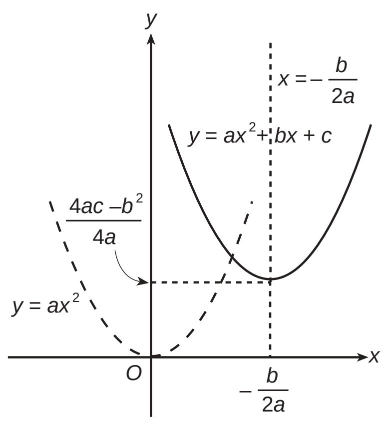


(a) $a>0$
- Curve opens upwards
- y attains minimum at $x=h$
- Line of symmetry is $x=h$

(b) $a<0$
- Curve opens downwards
- y attains maximum at $x=h$
- Line of symmetry is $x=h$

Functions and Absolute Values

Shape of $y = ax^2 + bx + c$ $(a \neq 0)$



$$y = ax^2 + bx + c$$

$$= a\left[x - \left(-\frac{b}{2a}\right)\right]^2 + \frac{4ac - b^2}{4a}$$

(a) $a > 0$
- Curve opens upwards
- y attains minimum at $x = -\frac{b}{2a}$
- Line of symmetry is $x = -\frac{b}{2a}$

(b) $a < 0$
- Curve opens downwards
- y attains maximum at $x = -\frac{b}{2a}$
- Line of symmetry is $x = -\frac{b}{2a}$

Absolute values

Definition of absolute value

$$|x| = \begin{cases} x & \text{if } x \geq 0 \\ -x & \text{if } x < 0 \end{cases}$$

Properties of absolute value

(a) $|x| \leq 0$

(b) $|x| = |-x|$

(c) $|xy| = |x||y|$

(d) $\left|\frac{x}{y}\right| = \frac{|x|}{|y|}$ where $y \neq 0$

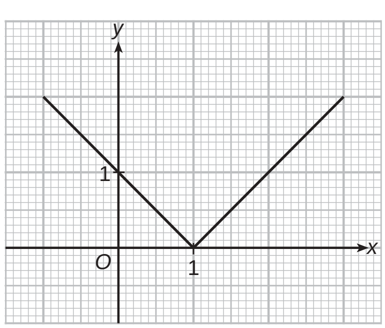
(e) $|x^2| = x^2 = |x|^2$

(f) If $a \geq 0$, then $|x| = a$ means $x = a$ or $x = -a$
If $a < 0$, then $|x| = a$ has no solutions

(g) $|x| = |y|$ means $x = y$ or $x = -y$

Graph of functions involving absolute value

- $y = |x - 1|$



1.1 Quadratic equations（二次方程）

Learning Focus

- Study the methods of solving the quadratic equation $ax^2 + bx + c = 0$ with $a \neq 0$.
- Determine the nature of roots of $ax^2 + bx + c = 0$ by discriminant.
- Study and apply the formulae of the sum and product of roots of the quadratic equation.
- Study the methods of the formation of quadratic equations.

A. Methods of solving quadratic equations

(a) Factorization（因式分解）

- Try to reduce the quadratic equation $ax^2 + bx + c = 0$ with $a \neq 0$ to form $(mx + n)(px + q) = 0$.

  Hence, the roots are $x = -\dfrac{n}{m}$ and $-\dfrac{q}{p}$.

(b) Quadratic formula（二次公式）

- The roots of the quadratic equation $ax^2 + bx + c = 0$ with $a \neq 0$ are given by

  $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Guided Example 1

Solve $(x^2 + 3x)^2 - 3(x^2 + 3x) - 4 = 0$.

Suggested Solution

$(x^2 + 3x)^2 - 3(x^2 + 3x) - 4 = 0$

Let $y = x^2 + 3x$ .... (1)

$\therefore \quad y^2 - 3y - 4 = 0$

$(y - 4)(y + 1) = 0$

$y = 4$ or $y = -1$ .... (2)

Put (2) into (1):

$x^2 + 3x = 4$ or $x^2 + 3x = -1$

$x^2 + 3x - 4 = 0$ or $x^2 + 3x + 1 = 0$

$(x - 1)(x + 4) = 0$ or $x = \dfrac{-3 \pm \sqrt{3^2 - 4(1)(1)}}{2(1)}$

$x = 1$ or -4 or $\dfrac{-3 \pm \sqrt{5}}{2}$

Reminder

In solving quadratic equation, there are two major methods: factorization and quadratic formula.

A. Family of parallel straight lines（平行綫族）

- If m is a constant, then the lines L: $y = mx + k$ represents a family of parallel straight lines with slope m as k varies.

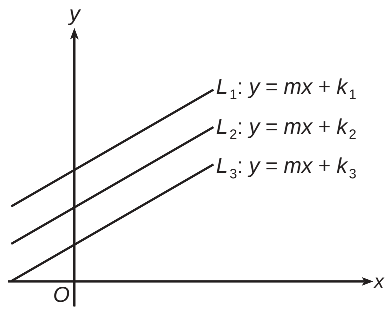


Figure 7.23

Reminder
As k varies, the straight line will have different positions but their slope are the same.

- If A and B are given constants, and k is real, then the lines L: $Ax + By + k = 0$ represents a family of parallel straight lines with slope equal to $-\dfrac{A}{B}$ as k varies.

B. Family of straight lines passing through the point of intersection of two given straight lines

- Given two straight lines L_1: $A_1x + B_1y + C_1 = 0$ and L_2: $A_2x + B_2y + C_2 = 0$ intersect at a point P. The line L: $(A_1x + B_1y + C_1) + k(A_2x + B_2y + C_2) = 0$, where k is real, represents a family of straight lines passing through the point P as k varies.

Reminder
By varying the value of k, the straight line obtained will have different slopes but will all pass through the point P.

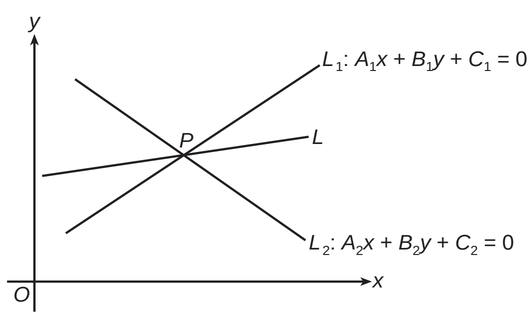


Figure 7.24

- L represents all straight lines passing through P except L_2.
- By similar argument, the lines L': $k(A_1x + B_1y + C_1) + (A_2x + B_2y + C_2) = 0$ represents a family of straight lines passing through the point P as k varies. L' represents all straight lines passing through P except L_1.

Guided Example 14

Find the equation of the two circles, both have centre at (8, 5) and touch the circle C: $x^2 + y^2 - 2y - 4 = 0$.

Suggested Solution

Centre of $C = (0, 1)$

Radius of $C = \sqrt{(0)^2 + 1^2 - (-4)} = \sqrt{5}$

Hence, graphically we have

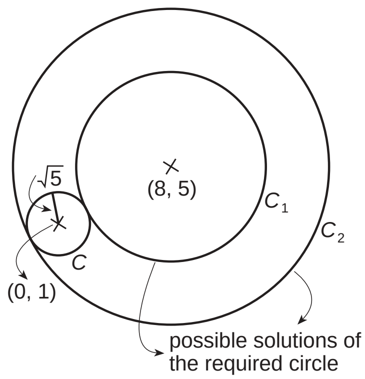


Figure 8.26

Let r_1 be the radius of circle C_1.

$r_1 + \sqrt{5}$ = Distance between (0, 1) and (8, 5)

$\therefore \quad r_1 + \sqrt{5} = \sqrt{(0-8)^2 + (1-5)^2}$

$= 4\sqrt{5}$

$\therefore \quad r_1 = 3\sqrt{5}$

Equation of C_1 is $(x-8)^2 + (y-5)^2 = (3\sqrt{5})^2$.

i.e. $x^2 + y^2 - 16x - 10y + 44 = 0$

Similarly, let r_2 be the radius of circle C_2.

$r_2 - \sqrt{5}$ = Distance between (0, 1) and (8, 5)

$\therefore \quad r_2 - \sqrt{5} = \sqrt{(0-8)^2 + (1-5)^2}$

$= 4\sqrt{5}$

$\therefore \quad r_2 = 5\sqrt{5}$

Equation of C_2 is $(x-8)^2 + (y-5)^2 = (5\sqrt{5})^2$

i.e. $\underline{\underline{x^2 + y^2 - 16x - 10y - 36 = 0}}$

Reminder

C and C_1 are connected externally, i.e.

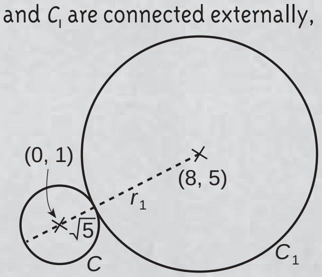


Figure 8.27

Reminder

C and C_2 are connected internally, i.e.

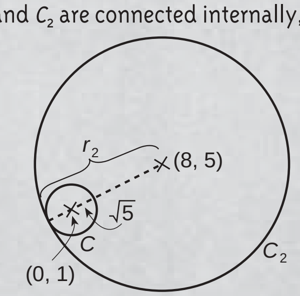


Figure 8.28

Glossary

angle between two planes	兩平面的夾角	inclination	傾角
angle of depression	俯角	line of greatest slope	最大斜率的直綫
angle of elevation	仰角	projection	投影
compass bearing	羅盤方位角	Sine Law	正弦公式
Cosine Law	餘弦公式	true bearing	真方位角

Important Formulae

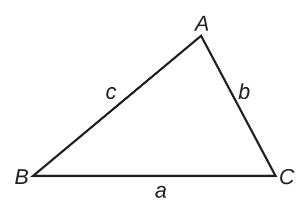
- The Sine Law

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

- The Cosine Law

$$a^2 = b^2 + c^2 - 2bc \cos A$$
$$b^2 = a^2 + c^2 - 2ac \cos B$$
$$c^2 = a^2 + b^2 - 2ab \cos C$$

or
$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$
$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$



Examination Question Analysis

Topics	Section A	Section B
Two-dimensional and three-dimensional problems	93(II–7), 95(II–7)	94(II–12), 96(II–12), 97(II–12), 98(II–13), 99(II–11), 00(II–12), 01(15), 02(17), 03(18)

Demonstration

Section A

1. In Figure 6.29, ΔABC is an isosceles triangle. $CA = CB$, $AB = 5$, $\angle ACD = 2\theta$ and $\angle BCD = \theta$.

   (a) Let $BD = x$. Using the Sine Law or otherwise, prove that $x = \dfrac{5}{1+2\cos\theta}$

   (b) As θ varies, prove that $\dfrac{5}{3} < x < \dfrac{5}{2}$. (7 marks)

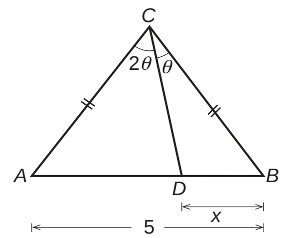


Figure 6.29

Guidelines

Note that $0° < \angle ACB < 180°$.

$\therefore\ 0° < 2\theta + \theta < 180°$.

Suggested Solution

(a) Consider ΔADC,

$$\frac{5-x}{\sin 2\theta} = \frac{CA}{\sin(180° - \beta)} \quad \text{............ (1)} \qquad \text{1A}$$

Consider ΔBDC,

$$\frac{x}{\sin\theta} = \frac{CB}{\sin\beta} \quad \text{............................. (2)} \qquad \text{1A}$$

Since $CA = CB$ and $\sin(180° - \beta) = \sin\beta$,

we have $\dfrac{CA}{\sin(180° - \beta)} = \dfrac{CB}{\sin\beta}$

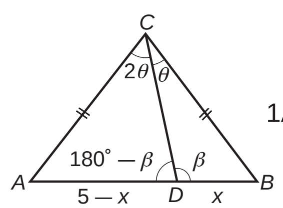


Figure 6.30

Guidelines

Note that

$\sin\angle ADC = \sin(180° - \angle CDB)$

$= \sin\angle CDB$

$\therefore$ We have

$$\frac{CA}{\sin\angle ADC} = \frac{CB}{\sin\angle CDB}.$$

Hence $\dfrac{5-x}{\sin 2\theta} = \dfrac{x}{\sin\theta}$ 1M

$$\frac{5-x}{2\sin\theta\cos\theta} = \frac{x}{\sin\theta}$$

$$5 - x = 2x\cos\theta$$

$$5 = x(1 + 2\cos\theta)$$

$$x = \frac{5}{1+2\cos\theta} \qquad \text{1A}$$

(b) Since

$$0° < \angle ACB < 180°$$

$$0° < 3\theta < 180°$$

$$0° < \theta < 60° \qquad \text{1A}$$

$$\cos 0° > \cos\theta > \cos 60°$$

$$1 > \cos\theta > \frac{1}{2}$$

$$3 > 1 + 2\cos\theta > 2 \qquad \text{1M}$$

$$\frac{5}{3} < \frac{5}{1+2\cos\theta} < \frac{5}{2} \qquad \text{1M}$$

$$\frac{5}{3} < x < \frac{5}{2}$$

Guidelines

The following inequalities are useful in solving part (b).

(i) If $0 < \theta_1 < \theta < \theta_2 < \dfrac{\pi}{2}$, then $\cos\theta_1 > \cos\theta > \cos\theta_2$; and

(ii) if $0 < a < b$, then $\dfrac{1}{a} > \dfrac{1}{b}$.

Practice

Unless otherwise specified, each numerical answer should be in exact value or correct to 3 significant figures.

Section A

1. In ΔABC (see Figure 6.48), $DE \parallel BC$. $DB = 6$ cm, $EC = 7$ cm and $\angle ABC = 72°$. Find $\angle BAC$.

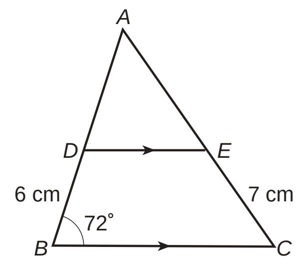


Figure 6.48

2. In quadrilateral $ABCD$ (see Figure 6.49), $AB = 6$, $BC = 5$, $CD = 8$, $\angle ABC = 120°$ and $\angle BCD = 100°$. Find AD. Hint 1

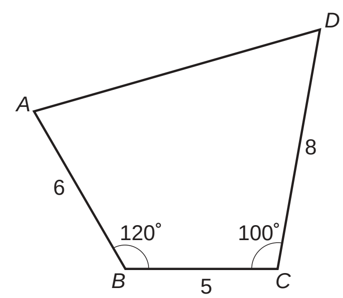


Figure 6.49

3. Solve ΔABC where $b = 7$, $c = 11$ and $\angle B = 34°$

4. In Figure 6.50, D is a point on BC such that AD bisects $\angle BAC$.
   (a) By considering the areas of ΔABD, ΔADC and ΔABC, or otherwise, prove that $\cos\theta = \dfrac{a(b+c)}{2bc}$.
   (b) Find the value of θ, correct to the nearest degree if $a = 2$, $b = 6$ and $c = 3$.

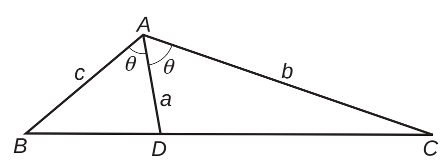


Figure 6.50

5. In ΔABC, if $\sin A : \sin B : \sin C = 2 : 5 : 6$. Hint 2
   (a) Find $\cos A$, $\cos B$ and $\cos C$.
   (b) Hence, find $\sin 2A : \sin 2B : \sin 2C$.

Index

1 Quadratic Equations, Quadratic Functions and Absolute Values

Section A

1. $(x^2 - x)^2 + 2(x^2 - x) - 3 = 0$

Let $y = x^2 - x$

$\therefore \quad y^2 + 2y - 3 = 0$

$(y + 3)(y - 1) = 0$

$y = -3 \quad \text{or} \quad y = 1$

$x^2 - x = -3 \quad \text{or} \quad x^2 - x = 1$

$x^2 - x + 3 = 0 \quad \text{or} \quad x^2 - x - 1 = 0$

$\Delta = (-1)^2 - 4(1)(3) = -11 < 0$

$\therefore$ No real roots.

$x = \dfrac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-1)}}{2(1)} = \dfrac{1 \pm \sqrt{5}}{2}$

2. $\sqrt{x+1} + \sqrt{3x-8} = 3$

$\sqrt{3x-8} = 3 - \sqrt{x+1}$

$\left(\sqrt{3x-8}\right)^2 = \left(3 - \sqrt{x+1}\right)^2$

$3x - 8 = 9 - 6\sqrt{x+1} + x + 1$

$2x - 18 = -6\sqrt{x+1}$

$x - 9 = -3\sqrt{x+1}$

$(x - 9)^2 = \left(-3\sqrt{x+1}\right)^2$

$x^2 - 18x + 81 = 9(x + 1)$

$x^2 - 27x + 72 = 0$

$(x - 3)(x - 24) = 0$

$x = 3 \quad \text{or} \quad 24 \text{ (rejected)}$

Reminder

In solving irrational equations, students should check the solutions by putting them back to the equation.

3. (a) $y(y - 1) - 2$

$y^2 - y - 2 = 0$

$(y - 2)(y + 1) = 0$

$y = 2 \text{ or } -1$

(b) $x^2 + x - 1 = \dfrac{2}{x^2 + x}$

Let $y = x^2 + x$

$y - 1 = \dfrac{2}{y}$

$y(y - 1) = 2$

$y = 2 \quad \text{or} \quad y = -1 \text{ (by (a))}$

$\therefore \quad x^2 + x = 2 \quad \text{or} \quad x^2 + x = -1$

$x^2 + x - 2 = 0 \quad\quad x^2 + x + 1 = 0$

$(x + 2)(x - 1) = 0$

$x = -2 \text{ or } 1$

$\Delta = 1^2 - 4(1)(1) = -3 < 0$

$\therefore$ No real roots.

Reminder

Use Δ to check the nature of roots.

4. (a) $\sqrt{x} + \dfrac{1}{\sqrt{x}} = \dfrac{5}{2}$

$\dfrac{x+1}{\sqrt{x}} = \dfrac{5}{2}$

$2(x + 1) = 5\sqrt{x}$

$[2(x + 1)]^2 = \left(5\sqrt{x}\right)^2$

$4x^2 + 8x + 4 = 25x$

$4x^2 - 17x + 4 = 0$

$(4x - 1)(x - 4) = 0$

$x = \dfrac{1}{4} \quad \text{or} \quad 4$

(b) $\sqrt{\dfrac{x+2}{x-1}} + \sqrt{\dfrac{x-1}{x+2}} = \dfrac{5}{2}$

Let $y = \dfrac{x+2}{x-1}$

$\sqrt{y} + \dfrac{1}{\sqrt{y}} = \dfrac{5}{2}$

By (a), $y = \dfrac{1}{4} \quad \text{or} \quad 4$

$\dfrac{x+2}{x-1} = \dfrac{1}{4} \quad \text{or} \quad \dfrac{x+2}{x-1} = 4$

$4x + 8 = x - 1 \quad \text{or} \quad x + 2 = 4x - 4$

$3x = -9 \quad\quad 6 = 3x$

$x = -3 \quad\quad x = 2$