Comparison between NEW and OLD syllabuses

The NEW Additional Mathematics syllabus is extracted from the old one. No new topics are added, but some topics are cut or trimmed. All the contents about 'Complex Numbers' as well as 'Conic Sections', such as ellipse, parabola and hyperbola had been removed from the syllabus. The changes of the topics of the new syllabus are listed in the following table:

Chapters	Topics of the syllabus	Topics removed
1. Quadratic Equations, Quadratic Functions and Absolute Values	• Quadratic functions and quadratic equations • Discriminant and nature of roots • Use of the absolute value sign	— — • Use of absolute value sign in relation to inequalities is not required.
2. Inequalities	• Quadratic inequalities in one variable	• Inequalities of the form $\frac{ax+b}{cx+d} \geq k$ are not required
3. Mathematical Induction	• Mathematical induction and its simple applications	—
4. Binomial Theorem	• The binomial theorem for positive integral indices	—
5. Trigonometry	• The six trigonometric functions of angles of any magnitude and their graphs • Formulae for $\sin(A \pm B)$, $\cos(A \pm B)$ and $\tan(A \pm B)$, sum and product formulae • General solution of simple trigonometric equations	— • Students are not required to prove these formulae. Their applications to multiple and half angles are expected but students are not required to memorize 'triple angle formulae' and 'half angle formulae' —
6. Solution of Triangles and its Applications	• Trigonometric problems in two- and three-dimensions	—

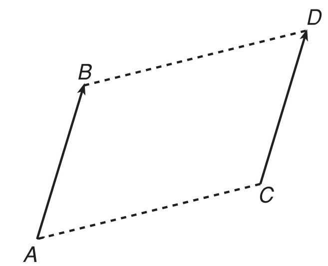
9 Vectors

Concept Map

Vectors

Basic concepts of vectors

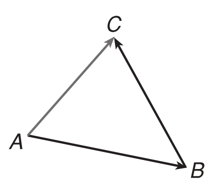
- A vector $\overrightarrow{AB}$ is represented by a directed line segment from an initial point A to a terminal point B.
- The magnitude(or length) of $\overrightarrow{AB}$ is denoted by $|\overrightarrow{AB}|$.
- If $AB = CD$ and $AB \,//\, CD$, then $\overrightarrow{AB} = \overrightarrow{CD}$.



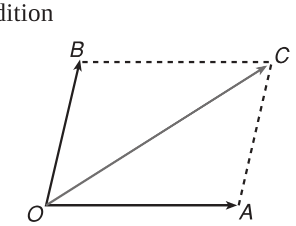
Vector operations

Addition, subtraction and scalar multiplication

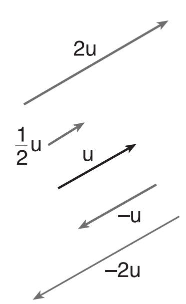
- Triangle law of addition
  $\overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC}$



- Parallelogram law of addition
  $\overrightarrow{OA} + \overrightarrow{OB} = \overrightarrow{OC}$



- Subtraction
  $\mathbf{u} - \mathbf{v} = \mathbf{u} + (-\mathbf{v})$.
- Scalar multiplication



Unit vector

unit vector of a non-zero vector $\mathbf{v}$ is

$\hat{\mathbf{v}} = \dfrac{\mathbf{v}}{|\mathbf{v}|}$ (or $\dfrac{1}{|\mathbf{v}|}\mathbf{v}$)

Rules of vectors

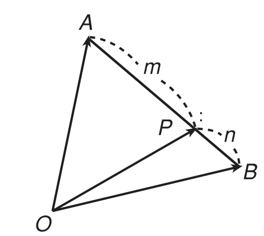
- $\mathbf{a} + \mathbf{b} = \mathbf{b} + \mathbf{a}$
- $(\mathbf{a} + \mathbf{b}) + \mathbf{c} = \mathbf{a} + (\mathbf{b} + \mathbf{c})$
- $m(n\mathbf{a}) = (mn)\mathbf{a} = n(m\mathbf{a})$
- $(m + n)\mathbf{a} = m\mathbf{a} + n\mathbf{a}$
- $m(\mathbf{a} + \mathbf{b}) = m\mathbf{a} + m\mathbf{b}$

Properties of vectors

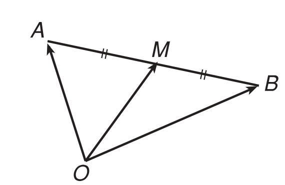
- $\mathbf{u} = k\mathbf{v}$, if and only if $\mathbf{u} \,//\, \mathbf{v}$.
- For two non-zero and non-parallel vectors $\mathbf{u}$ and $\mathbf{v}$,
  - if $k_1\mathbf{u} + k_2\mathbf{v} = \mathbf{0}$, then $k_1 = k_2 = 0$.
  - if $m_1\mathbf{u} + n_1\mathbf{v} = m_2\mathbf{u} + n_2\mathbf{v}$, then $m_1 = m_2$ and $n_1 = n_2$.
  - if $(m_1\mathbf{u} + n_1\mathbf{v}) \,//\, (m_2\mathbf{u} + n_2\mathbf{v})$, then $\dfrac{m_1}{m_2} = \dfrac{n_1}{n_2}$.

Point of division

$\overrightarrow{OP} = \dfrac{n\overrightarrow{OA} + m\overrightarrow{OB}}{m + n}$



$\overrightarrow{OM} = \dfrac{\overrightarrow{OA} + \overrightarrow{OB}}{2}$

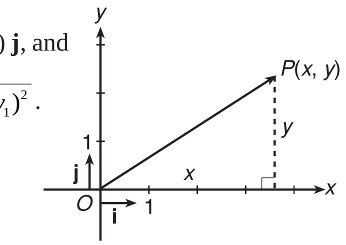


Vectors in Cartesian plane

If $A(x_1, y_1)$ and $B(x_2, y_2)$ be two points in the Cartesian plane, we have

$\overrightarrow{AB} = (x_2 - x_1)\,\mathbf{i} + (y_2 - y_1)\,\mathbf{j}$, and

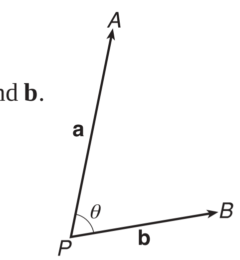
$|\overrightarrow{AB}| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.



Scalar product

Definition of scalar product of two vectors

$\mathbf{a} \cdot \mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos\theta$,
where $\theta (0^\circ \le \theta \le 180^\circ)$
is the angle between $\mathbf{a}$ and $\mathbf{b}$.



Properties of scalar product

- $\mathbf{a} \cdot \mathbf{b} = \mathbf{b} \cdot \mathbf{a}$
- $k(\mathbf{a} \cdot \mathbf{b}) = (k\mathbf{a}) \cdot \mathbf{b} = \mathbf{a} \cdot (k\mathbf{b})$
- $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c}) = \mathbf{a} \cdot \mathbf{b} + \mathbf{a} \cdot \mathbf{c}$
- $\mathbf{a} \cdot \mathbf{a} = |\mathbf{a}|^2$
- Given any non-zero vectors $\mathbf{a}$ and $\mathbf{b}$. $\mathbf{a} \perp \mathbf{b}$ if and only if $\mathbf{a} \cdot \mathbf{b} = 0$

Scalar product in Cartesian plane

- $\mathbf{i} \cdot \mathbf{i} = \mathbf{j} \cdot \mathbf{j} = 1$, $\mathbf{i} \cdot \mathbf{j} = \mathbf{j} \cdot \mathbf{i} = 0$
- If $\mathbf{a} = x_1\mathbf{i} + y_1\mathbf{j}$ and $\mathbf{b} = x_2\mathbf{i} + y_2\mathbf{j}$, then $\mathbf{a} \cdot \mathbf{b} = x_1x_2 + y_1y_2$.
- If $\mathbf{a}$ and $\mathbf{b}$ are non-zero vectors and the angle between $\mathbf{a}$ and $\mathbf{b}$ is θ, then

$$\cos\theta = \frac{x_1x_2 + y_1y_2}{\sqrt{x_1^2 + y_1^2}\sqrt{x_2^2 + y_2^2}}.$$

9.1 Vector operations

Learning Focus

- Understand the basic concepts of vectors.
- Study the method of addition and subtraction of vectors.
- Learn the method of scalar multiplication of vectors.
- Apply the rules of operations of vectors to solve problems.

A. Basic concepts of vectors

- A scalar（標量／純量）is a quantity possesses magnitude only.
  A vector（向量／矢量）is a quantity possesses both direction and magnitude.
- Geometrically, a vector $\overrightarrow{AB}$ is represented by a directed line segment from an initial point A to a terminal point B (see Figure 9.1). The magnitude of $\overrightarrow{AB}$ is specified by the length of AB, and is denoted by $|\overrightarrow{AB}|$. The direction of $\overrightarrow{AB}$ is the direction from A to B.

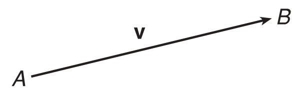


Figure 9.1

- Bold face letters can also be used to denote vectors.
  For example: Let $\mathbf{v} = \overrightarrow{AB}$. In this case, $|\mathbf{v}|$ is the magnitude of $\mathbf{v}$.

Reminder
It is not suggested to write a bold face **v** to represent vectors in doing exercises about vectors. Using the arrow $\vec{v}$ is more appropriate.

- Two vectors are equal if they have the same magnitude and direction. The two equal vectors need not have the same initial point and terminal point.
  For example, in the parallelogram $ABCD$ shown in Figure 9.2, we have $AB = CD$ and $AB \parallel CD$, therefore $\overrightarrow{AB} = \overrightarrow{CD}$.

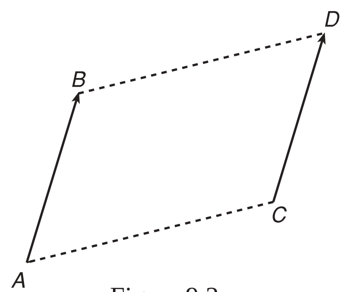


Figure 9.2

- A vector with zero magnitude is called a zero vector（零向量）and is denoted by $\mathbf{0}$. A zero vector has no specific direction.
  For example, $\overrightarrow{AA}$ is a vector from the point A to A. Since the length of AA is zero, we have $\overrightarrow{AA} = \mathbf{0}$.

B. Addition of vectors

- Triangle law of addition（三角形加法律）:
  If ABC is a triangle, then $\overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{AC}$.

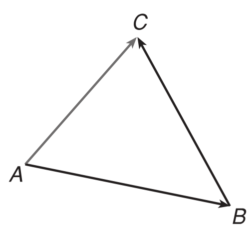


Figure 9.3

Demonstration

Section A

1. $\overrightarrow{OA} = \mathbf{i} + 3\mathbf{j}$, $\overrightarrow{OB} = 3\mathbf{i} - \mathbf{j}$, C is a point on AB such that $\dfrac{AC}{CB} = r$.

Guidelines: Using the formula of point of division.

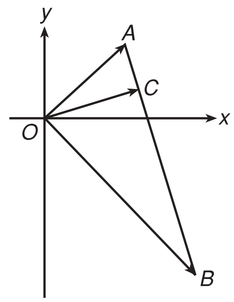


Figure 9.31

(a) Express $\overrightarrow{OC}$ in terms of r.

(b) Find the value of r if OC is perpendicular to AB. Hence find the coordinates of C. (6 marks)

Guidelines: Students should employ scalar product to prove two lines are perpendicular to each other in the problems of vectors.

Suggested Solution

(a) By point of division,

$$\overrightarrow{OC} = \frac{(1)(\overrightarrow{OA}) + r(\overrightarrow{OB})}{1+r}$$

$$= \frac{(\mathbf{i} + 3\mathbf{j}) + r(3\mathbf{i} - \mathbf{j})}{1+r} \quad \text{1A}$$

$$= \left(\frac{1+3r}{1+r}\right)\mathbf{i} + \left(\frac{3-r}{1+r}\right)\mathbf{j} \quad \text{1A}$$

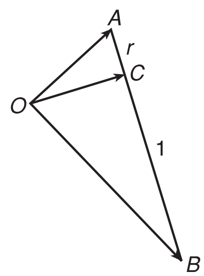


Figure 9.32

(b) If $OC \perp AB$,

$$\overrightarrow{OC} \cdot \overrightarrow{AB} = 0$$

$$\therefore \left[\left(\frac{1+3r}{1+r}\right)\mathbf{i} + \left(\frac{3-r}{1+r}\right)\mathbf{j}\right] \cdot (\overrightarrow{OB} - \overrightarrow{OA}) = 0 \quad \text{1M}$$

$$\therefore \left[\left(\frac{1+3r}{1+r}\right)\mathbf{i} + \left(\frac{3-r}{1+r}\right)\mathbf{j}\right] \cdot (2\mathbf{i} - 4\mathbf{j}) = 0$$

$$\therefore \left(\frac{1+3r}{1+r}\right)(2) + \left(\frac{3-r}{1+r}\right)(-4) = 0 \quad \text{1M}$$

$$r = 1 \quad \text{1A}$$

Hence $\overrightarrow{OC} = \left[\dfrac{1+3(1)}{1+1}\right]\mathbf{i} + \left[\dfrac{3-(1)}{1+1}\right]\mathbf{j}$

$= 2\mathbf{i} + \mathbf{j}$

$\therefore$ The coordinates of C are (2, 1). 1A

Practice

Section A

1. In ΔABC, D is the mid-point of BC. Prove that $2\overrightarrow{CA} + 3\overrightarrow{BC} + 4\overrightarrow{AB} = 2\overrightarrow{AD}$.

2. Given that **a**, **b** are non-zero, non-parallel vectors such that

$$\frac{rs}{r+1}\mathbf{a} + s\mathbf{b} = \frac{2-s}{3}\mathbf{a} + \frac{r}{r+1}\mathbf{b}.$$

Find r and s. Hint 1

3. (a) If $x\mathbf{i} + y\mathbf{j}$ is perpendicular to $5\mathbf{i} + 12\mathbf{j}$, find the ratio of x to y.

   (b) Hence, or otherwise, find the unit vector(s) perpendicular to $5\mathbf{i} + 12\mathbf{j}$.

4. In the figure, $|\overrightarrow{AB}| = \sqrt{3}, |\overrightarrow{AC}| = 2$ and $\angle CAB = \frac{\pi}{6}$.

   (a) Find $\overrightarrow{AB} \cdot \overrightarrow{AC}$.

   (b) Find $\left|\overrightarrow{AB} + 3\overrightarrow{AC}\right|$. Hint 2

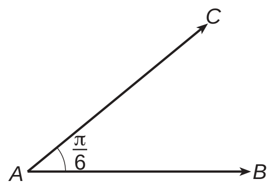


Figure 9.42

5. Given that $\overrightarrow{OP} = 2\mathbf{i} + 4\mathbf{j}$, $\overrightarrow{OQ} = \mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{OR} = k\mathbf{i} + 4\mathbf{j}$.

   (a) Find $\overrightarrow{QP}$ and $\overrightarrow{QR}$.

   (b) If $\angle PQR = 120°$, find the values of k.

6. $ABCD$ is a parallelogram. $DE = 2EC$, BD is the diagonal and $BF = r\,DF$. Suppose $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AD} = \mathbf{d}$, $AF = s\,AE$.

   (a) Find $\overrightarrow{AF}$ in terms of **a**, **d** and r.

   (b) Find $\overrightarrow{AF}$ in terms of **a**, **d**, and s. Hint 3

   (c) Hence, or otherwise, find the values of r and s.

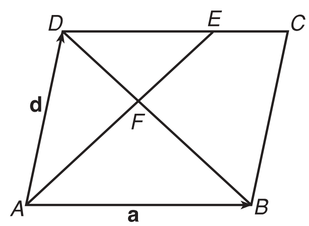


Figure 9.43

7. Given that $\overrightarrow{AB} = \mathbf{a}$, $\overrightarrow{AF} = \mathbf{b}$. $\frac{CE}{EB} = \frac{2}{3}$ and $\frac{DE}{EF} = \frac{5}{4}$.

   Let $\overrightarrow{AD} = r\mathbf{a}$ and $\overrightarrow{AC} = s\mathbf{b}$.

   (a) Express $\overrightarrow{AE}$ in terms of r, **a** and **b** and express $\overrightarrow{AE}$ in terms of s, **a** and **b**.

   (b) Find r and s.

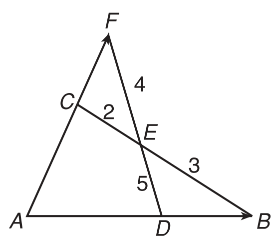


Figure 9.44

10.1 Derivative from first principles

Learning Focus

- Understand the concepts of limits of functions.
- Learn the theorems and techniques of finding limits of functions.
- Learn how to find the slope of a function.
- Understand the definition of the derivative of a function (the first principle).
- Apply the first principle to find the derivatives of functions.

A. Basic concepts of limits of functions

- A function $f(x)$ is said to have a limit（極限）L as x approaches a if $f(x)$ can be made as close to a finite number L as we like by taking x sufficiently close to a.
- In symbols, we write $\lim_{x \to a} f(x) = L$.
- For example,
  (i) if $x \to 2$, then $x + 3 \to 5$. So we write $\lim_{x \to 2}(x + 3) = 5$.
  (ii) If $x \to -3$, then $x^2 \to 9$. So we write $\lim_{x \to -3} x^2 = 9$.

Reminder
Note that $x \to a$ means x approaches a.

B. Theorems on limits of functions

- Let $\lim_{x \to a} f(x) = A$, $\lim_{x \to a} g(x) = B$, where A and B are finite numbers, and k is a real constant. Then we have:

(i) $\lim_{x \to a} k = k$

(ii) $\lim_{x \to a} kf(x) = k \lim_{x \to a} f(x) = kA$

(iii) $\lim_{x \to a}[f(x) \pm g(x)] = \lim_{x \to a} f(x) \pm \lim_{x \to a} g(x) = A \pm B$

(iv) $\lim_{x \to a}[f(x) \cdot g(x)] = \lim_{x \to a} f(x) \cdot \lim_{x \to a} g(x) = A \cdot B$

(v) $\lim_{x \to a} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x \to a} f(x)}{\lim_{x \to a} g(x)} = \dfrac{A}{B}$, provided that $B \neq 0$.

(vi) $\lim_{x \to a} \sqrt{f(x)} = \sqrt{\lim_{x \to a} f(x)} = \sqrt{A}$, provided that $A \geq 0$.

C. Useful formulae

(i) $\lim_{x\to\infty} \frac{1}{x} = 0$

(ii) $\lim_{\theta\to 0} \frac{\sin\theta}{\theta} = \lim_{\theta\to 0} \frac{\tan\theta}{\theta} = 1$, where θ is in radian measure.

D. Slope of the tangent to a curve

- In Figure 10.1, $A(x_0, f(x_0))$ is a fixed point and $B(x_0 + \Delta x, f(x_0 + \Delta x))$ is a variable point on the curve.
  AT is a tangent（切線）to the curve at A.

$$\text{Slope of } AB = \frac{\Delta y}{\Delta x} = \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

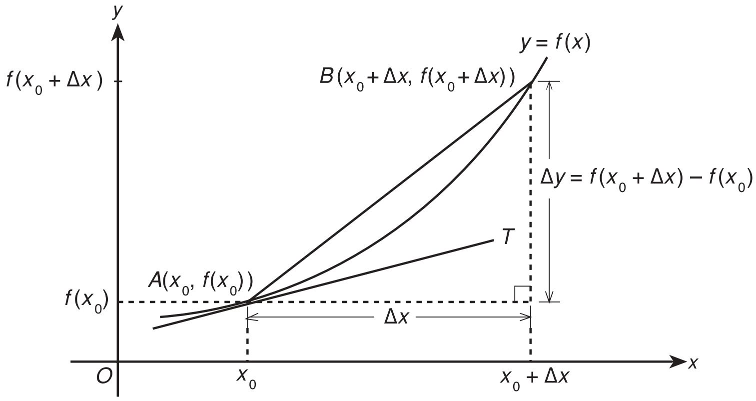


Figure 10.1

- When B gets closer and closer to A, Δx approaches zero and the slope of AB approaches the slope of AT. Thus

$$\text{Slope of } AT = \lim_{\Delta x\to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x\to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}.$$

- We take the slope of this tangent AT as the slope of the curve at A.

Reminder

Δx is called the increment（增量）of x.

E. Derivatives

- The derivative（導數）of a function $y = f(x)$ with respect to x, denoted by $\frac{dy}{dx}$, is defined as $\frac{dy}{dx} = \lim_{\Delta x\to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$, provided that this limit exists.
- The above definition is called first principles（基本原理）of finding the derivative of a function of $f(x)$ with respect x.
- $\frac{dy}{dx}$ may be denoted as $f'(x)$, y' or $\frac{d}{dx} f(x)$ and is called the derived function（導函數）.
- The derivative of $f(x)$ at $x = x_0$ is the slope of the tangent to the curve $y = f(x)$ at $x = x_0$.
  We denote it by $\left.\frac{dy}{dx}\right|_{x=x_0}$ or $f'(x_0)$.
- The process of finding the derivative of a function is called differentiation（微分法）.

Reminder

If $\left.\frac{dy}{dx}\right|_{x=o}$ exists, the function $y = f(x)$ is said to be differentiable（可微的）at $x = x_0$.

Guided Example 11

Let C be the curve $y = x^3 - 3x + 1$. $P(1, -1)$ and $Q(-2, -1)$ are two points on C.

(a) Find the equations of the tangent and the normal to C at P.

(b) Show that the normal to C at Q passes through the point $B(7, -2)$.

Suggested Solution

(a) $\dfrac{dy}{dx} = 3x^2 - 3$

$\therefore$ Slope of the tangent at $(1, -1)$ $= \left.\dfrac{dy}{dx}\right|_{\substack{x=1 \\ y=-1}}$

$= 3(1)^2 - 3$

$= 0$

$\therefore$ The equation of the tangent is

$\underline{\underline{y = -1}}$.

$\therefore$ The equation of the normal is

$\underline{\underline{x = 1}}$.

(b) Slope of the tangent at $Q(-2, -1) = 3(-2)^2 - 3$

$= 9$

$\therefore$ Slope of the normal at $Q = (-2, -1) = -\dfrac{1}{9}$

Note that slope of $BQ = \dfrac{(-2) - (-1)}{7 - (-2)}$

$= -\dfrac{1}{9}$

$=$ slope of the normal at Q

$\therefore$ The normal to C at Q will pass through B.

Reminder

The slope of the tangent at P is 0 means that the tangent line is horizontal. Hence, the normal at P is vertical.

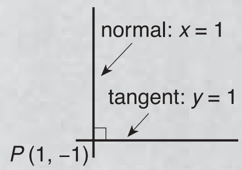


Figure 10.3

Glossary

Chain Rule	鏈式法則	limit	極限
derivative	導數	normal	法線
derived function	導函數	parameter	參數
differentiation	微分法	parametric function	參數函數
first derivative	一階導數	Power Rule	冪規律
first principles	基本原理	Product Rule	積法則
implicit function	隱函數	Quotient Rule	商法則
increment	增量	second derivative	二階導數
inverse function	反函數	tangent	切線

Guided Example 7

Figure 11.15 shows a right circular cone of radius x cm and height $(6 - x)$ cm. Let V cm^3 be the volume of the cone.

(a) (i) Express V in terms of x.
(ii) Hence find the range of values of x for which
(1) V is increasing, and
(2) V is decreasing.

(b) The cone is placed completely inside a right circular cylinder of radius 3 cm and height 4 cm, as shown in Figure 11.16.
(i) Show that $2 \le x \le 3$.
(ii) Hence find the greatest volume of the cone.

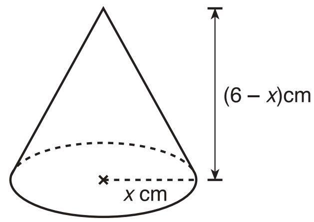


Figure 11.15

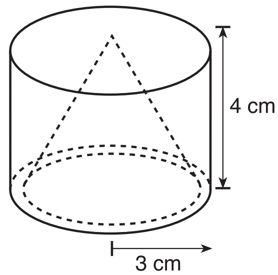


Figure 11.16

Suggested Solution

(a) (i) $V = \frac{1}{3}\pi x^2(6 - x)$

(ii) $\frac{dV}{dx} = \frac{1}{3}\pi(12x - 3x^2)$
$= \pi x(4 - x)$

x	$0 < x < 4$	$x = 4$	$4 < x < 6$
$\frac{dV}{dx}$	+	0	–

$\therefore$ For $0 < x < 4$, V is increasing.
For $4 < x < 6$, V is decreasing.

(b) (i) By comparing the height and the base radius of the cone and the cylinder, we have
$x \le 3$ and $6 - x \le 4$
$x \le 3$ and $x \ge 2$
$\therefore$ $2 \le x \le 3$

(ii) Since V is increasing for $2 \le x \le 3$, V is maximized when $x = 3$.
The greatest volume of the cone
$= \frac{1}{3}\pi(3)^2(6 - 3)$
$= 9\pi$ cm^3

Reminder

The radius and the height of the cone must be positive.
Hence $x > 0$ and $6 - x > 0$.
$\therefore$ $0 < x < 6$.

Reminder

From the graph of $V = \frac{1}{3}\pi x^2(6 - x)$, we can see that for $2 \le x \le 3$, V is maximum when $x = 3$.

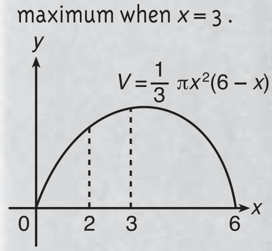


Figure 11.17

Important Formulae

- The First Derivative Test
  Let $y = f(x)$ be a differentiable function defined in an interval $a < x < b$.
  - If $\left.\frac{dy}{dx}\right|_{x=x_1} = 0$, and $\frac{dy}{dx}$ changes sign from + to – as x increases through x_1, then $(x_1, f(x_1))$ is a maximum point.
  - If $\left.\frac{dy}{dx}\right|_{x=x_2} = 0$ and $\frac{dy}{dx}$ changes sign from – to + as x increases through x_2, then $(x_2, f(x_2))$ is a minimum point.
- The Second Derivative Test
  Let $y = f(x)$ be a function which has a second derivative.
  - If $\left.\frac{dy}{dx}\right|_{x=x_1} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{x=x_1} < 0$, then $(x_1, f(x_1))$ is a maximum point of the curve $y = f(x)$.
  - If $\left.\frac{dy}{dx}\right|_{x=x_2} = 0$ and $\left.\frac{d^2y}{dx^2}\right|_{x=x_2} > 0$, then $(x_2, f(x_2))$ is a minimum point of the curve $y = f(x)$.
- If a quantity Q is a function of time t, then the derivative $\frac{dQ}{dt}$ is the rate of change of Q with respect to time t.
  - If $\frac{dQ}{dt} > 0$, then Q increases as t increases.
  - If $\frac{dQ}{dt} < 0$, then Q decreases as t increases.

Examination Question Analysis

Topics	Section A	Section B
Finding the greatest and least values	91(I – 4), 03(I – 13)	——
Rate of change	92(I – 7), 97(I – 4), 99(I – 8)	91(I – 12), 96(I – 11), 98(I – 13), 00(I – 13)
Curves sketching	95(I – 3)	90(I – 10), 91(I – 11), 92(I – 12), 93(I – 11), 94(I – 9), 96(I – 9), 97(I – 10), 98(I – 10), 99(I – 9), 00(I – 10), 01(18)
Optimization problems	——	90(I – 11), 92(I – 11), 93(I – 9), 94(I – 12), 95(I – 9), 95(I – 12), 97(I – 12), 97(I – 13), 99(I – 12), 99(I – 13), 02(14), 04(16)

Additional Mathematics

Mock Examination 1

Time Allowed: $2\frac{1}{2}$ hours

This paper must be answered in English

1. Answer **ALL** questions in Section A and any **FOUR** questions in Section B.
2. Write your answers in the answer book provided. **For Section A, there is no need to start each question on a fresh page.**
3. All working must be clearly shown.
4. Unless otherwise specified, numerical answers must be **exact.**
5. In this paper, vectors may be represented by bold-type letters such as **u**, but candidates are expected to use appropriate symbols such as $\vec{u}$ in their working.
6. The diagrams in the paper are not necessarily drawn to scale.

Index

10 Differentiation

Section A

1. (a)
$$\lim_{x\to 2}\frac{3x^2-5x-2}{x^2-6x+8}=\lim_{x\to 2}\frac{(3x+1)(x-2)}{(x-4)(x-2)}$$
$$=\lim_{x\to 2}\frac{3x+1}{x-4}$$
$$=\frac{3(2)+1}{(2)-4}$$
$$=\underline{\underline{-\frac{7}{2}}}$$

(b)
$$\lim_{x\to 1}\frac{(x-1)^3}{x^4-1}=\lim_{x\to 1}\frac{(x-1)^3}{(x^2-1)(x^2+1)}$$
$$=\lim_{x\to 1}\frac{(x-1)^3}{(x-1)(x+1)(x^2+1)}$$
$$=\lim_{x\to 1}\frac{(x-1)^2}{(x+1)(x^2+1)}$$
$$=\frac{((1)-1)^2}{((1)+1)((1)^2+1)}$$
$$=\underline{\underline{0}}$$

(c)
$$\lim_{x\to 0}\frac{\frac{1}{x+2}-\frac{1}{2}}{x}=\lim_{x\to 0}\frac{1}{x}\cdot\frac{2-x-2}{(x+2)2}$$
$$=\lim_{x\to 0}\frac{-1}{(x+2)2}$$
$$=\frac{-1}{((0)+2)2}$$
$$=\underline{\underline{-\frac{1}{4}}}$$

2. (a)
$$\lim_{x\to 1}\left(\frac{3}{x^3-1}-\frac{1}{x-1}\right)$$
$$=\lim_{x\to 1}\left[\frac{3}{(x-1)(x^2+x+1)}-\frac{1}{x-1}\right]$$
$$=\lim_{x\to 1}\frac{3-x^2-x-1}{(x-1)(x^2+x+1)}$$
$$=\lim_{x\to 1}\frac{-(x-1)(x+2)}{(x-1)(x^2+x+1)}$$
$$=\lim_{x\to 1}\frac{-(x+2)}{x^2+x+1}$$
$$=\frac{-((1)+2)}{(1)^2+(1)+1}$$
$$=\underline{\underline{-1}}$$

(b)
$$\lim_{x\to 2}\frac{3-\sqrt{x+7}}{x^2-4}$$
$$=\lim_{x\to 2}\frac{(3-\sqrt{x+7})(3+\sqrt{x+7})}{(x-2)(x+2)(3+\sqrt{x+7})}$$
$$=\lim_{x\to 2}\frac{9-x-7}{(x-2)(x+2)(3+\sqrt{x+7})}$$
$$=\lim_{x\to 2}\frac{-(x-2)}{(x-2)(x+2)(3+\sqrt{x+7})}$$
$$=\frac{-1}{(2+2)(3+\sqrt{2+7})}$$
$$=\underline{\underline{-\frac{1}{24}}}$$

3. (a)
$$\lim_{x\to 0}\frac{x}{\sqrt{1+x+x^2}-1}$$
$$=\lim_{x\to 0}\frac{x}{\sqrt{1+x+x^2}-1}\cdot\frac{\sqrt{1+x+x^2}+1}{\sqrt{1+x+x^2}+1}$$
$$=\lim_{x\to 0}\frac{x(\sqrt{1+x+x^2}+1)}{x+x^2}$$
$$=\lim_{x\to 0}\frac{\sqrt{1+x+x^2}+1}{1+x}$$
$$=\underline{\underline{2}}$$

(b)
$$\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}=\lim_{x\to 1}\frac{x-1}{\sqrt{x^2+3}-2}\cdot\frac{\sqrt{x^2+3}+2}{\sqrt{x^2+3}+2}$$
$$=\lim_{x\to 1}\frac{(x-1)\sqrt{x^2+3}+2}{x^2-1}$$
$$=\lim_{x\to 1}\frac{(x-1)(\sqrt{x^2+3}+2)}{(x-1)(x+1)}$$
$$=\lim_{x\to 1}\frac{\sqrt{x^2+3}+2}{x+1}$$
$$=\underline{\underline{2}}$$

4. (a)
$$\lim_{x\to\infty}\frac{4x^3-5x^2-1}{6x^3+7x-2}=\lim_{x\to\infty}\frac{4-\frac{5}{x}-\frac{1}{x^3}}{6+\frac{7}{x^2}-\frac{2}{x^3}}$$
$$=\frac{4}{6}$$
$$=\underline{\underline{\frac{2}{3}}}$$