Exam Paper Format

The HKCE Mathematics Examination, starting from 2006, consists of two papers.

	Paper 1			Paper 2	
Types of questions	Conventional questions			Multiple-choice questions	
Duration	2 hours			1 hour 30 minutes	
Percentage share of the total marks	60% 33 marks [Section A (1)] 33 marks [Section A (2)] 33 marks [Section B]			40% $\dfrac{2}{3}$ of the paper mark (Section A) $\dfrac{1}{3}$ of the paper mark (Section B)	
Details of the papers	<u>Section A (1)</u> • 8 to 10 compulsory short elementary questions from the foundation syllabus • Each question consists of 1 to 2 parts.	<u>Section A (2)</u> • 4 to 5 compulsory harder questions from the foundation syllabus • Each question consists of 2 to 4 parts	<u>Section B</u> • 3 out of 4 demanding questions from the whole syllabus • Each question consists of 3 to 5 parts	<u>Section A</u> • 36 compulsory questions from the foundation syllabus	<u>Section B</u> • 18 compulsory questions from the whole syllabus
				• Totally consists of 54 multiple-choice questions • Four options will be provided in each question.	

Topics \ Years	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004
Percentages	4	12	10a	7	17a	10b	8, 14b	6b	5	3
Estimation, Rates, Ratios, and Variations	5	13a	/	12	6	8	13a	11a	10a	10a
Polynomials and Formulas	1b, 2	1, 4	1	5,9	2, 15b	1,6	2,6	4	1, 3	2, 6
Indices, Surds and Logarithms	7	2	2, 3, 10b	4	1	2	1	1	4	1
Functions and Graphs	8	/	13	/	7	/	13c	/	/	4
Equations	6	4, 13a, 13b, 13c	8	/	15b	10a	13b, 16a	/	6	7
Inequalities and Linear Programming	1a, 12	5, 9b, 9c	4	18	3, 17b(ii)	5, 15	4, 15a, 15b(i)	17, 17b	2, 10b	10b
Trigonometry	6, 15	15,17b, 17c	6, 12a(i)	3, 17	4, 9a, 18	4, 17	9, 16b 17a(ii)	3, 14	9, 14, 15a(ii)	5, 17
Sequences	3	3	15	13	17b(i)	14	12a, 12b(ii)	13b, 13c	7, 15b(ii)	15
Mensuration	13a, 13c(i), (ii)	7a, 8, 16a	5c, 7a, 12a(ii) 12b(i), (ii), (iii)	1, 16a	9b, 13	3, 18a, 18b(i), (ii)	3, 9, 12b(i), 16a	2, 6a, 11b, 13a, 15a(i), 15b	13, 15(a)(i)	9, 12b(ii)
Deductive Geometry	1c, 1d, 13b	10		2, 6, 14	14	13	11	10	8, 15a(iii), 15b(i)	12a, 12b(i)
Circles	14	6	9, 16a	6, 14	5, 16a, 16b(iii)	7, 16a, 16b(i)	5, 17b	9, 16a	17a	16a, 16b, 16c(i)
Coordinate Geometry	10	9a, 11	16b	8, 15	10, 16b(i), (ii)	9, 16b(ii)	7, 17a(i)	8, 16b, 17a	12, 17b	13, 14a, 14c, 16c(ii)
Probability	11	7b	14	11	12	12	15b(ii)	12c	16	8
Statistics	9	14	11	10	8, 11	11	10	5, 12a, 12b	11	11

Exam Strategies

A. General Strategies

1. In the examination centre

- Make sure your watch matches with that of the examination centre.
- Listen carefully to the invigilator for any errors and changes in the examination papers.
- Read carefully the instructions on the cover of the answer book or question book.
- Check carefully whether there are any omitted or blank pages in the examination paper or not according to the invigilator's instruction.

2. During the examination

- Attempt the required number of questions. (Please refer to Exam Paper Format on p v.)
- Use proper stationeries.
  - Paper 1: use a pen mainly, but a HB pencil for drawing.
  - Paper 2: use a HB pencil.
- Show your work clearly and neatly.
- Do not be stuck in any one of the questions. Skip it and go on to another one.

3. After finish answering the questions

- Do not be tempted to leave early.
- Check whether there are any questions missed out or not.
- Go back to questions skipped earlier.
- Check whether there are any careless mistakes or not.
- Do not cross out anything before you find enough time to replace it correctly.
- Make sure you write your candidate number on the answer book, supplementary answer sheets and multiple-choice answer sheet.

B. Specific Strategies

1. Paper 1 (2 hours)

- Allocate a reasonable proportion of time to each section and allow 5 minutes for final checking.

Sections	Suggested Time Allocation	Approximate Time per Question
A (1)	30 minutes	3 – 4 minutes
A (2)	40 minutes	8 – 10 minutes
B	45 minutes	15 minutes

- Make your draft in the draft paper provided.
- Show your formulas and steps rather than just writing down the answers. In case you do not get the correct answer, you can get the marks for the correct methods used.

1 Percentages

Review

1.1 Percentages

- A percentage is a fraction with 100 as the denominator. For example, $\dfrac{44}{100}$.

- A percentage can be expressed by the notation %. For example, 44%.

- A percentage can be converted to a decimal or a fraction. For example, 44% = 0.44 or $\dfrac{11}{25}$.

1.2 Percentage increase and decrease

Percentage increase or decrease is used to describe the relative increase or decrease of the original value to the new value.
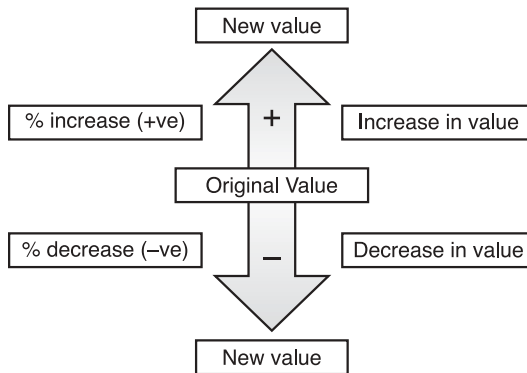
Figure 1.1

$$\text{Percentage increase} = \frac{\text{Value increased}}{\text{Original value}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{Value decreased}}{\text{Original value}} \times 100\%$$

$$\text{Percentage change} = \frac{\text{New value} - \text{Original value}}{\text{Original value}} \times 100\%$$

For example, Denny's pocket money is changed from $40 to $50.

$$\text{Percentage change} = \frac{50 - 40}{40} \times 100\% = 25\%$$

Reminder

A positive percentage change corresponds to an increase in value and a negative percentage change corresponds to a decrease in value.

1.6 Growth and depreciation

Let P be the original value, $r\%$ be the growth or depreciation rate and n be the number of period.
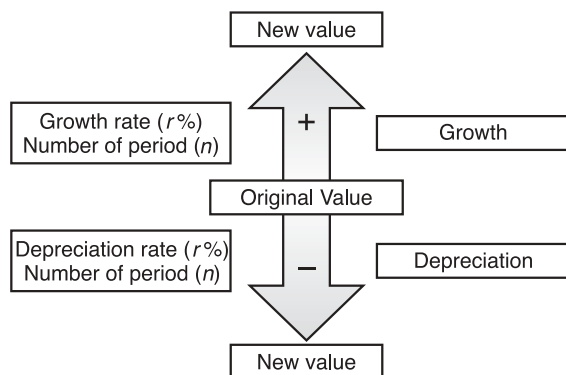

Figure 1.4

> New value after growth $= P \times (1 + r\%)^n$
>
> Growth $=$ New value $- P$
>
> New value after depreciation $= P \times (1 - r\%)^n$
>
> Depreciation $= P -$ New value

Reminder

$(1 + r\%)$ is called the growth factor. $(1 - r\%)$ is called the decay factor.

For example, the value of a new car is \$130 000. If the monthly depreciation rate is 1.5%, then after a year,

New value $= \$130\,000(1 - 1.5\%)^{12} = \$108\,437$

Depreciation $= \$130\,000 - \$108\,437 = \$21\,563$

1.7 Successive percentage change

Successive percentage change is the different percentage changes at different successive stages of an amount.

For example, the population of a city was 28 000 in 2003. From 2003 to 2004, it was increased by 5%. From 2004 to 2005, it was decreased by 8%.

Population in 2005 $= 28\,000(1 + 5\%)(1 - 8\%) = 27\,048$

Percentage change $= \dfrac{27\,048 - 28\,000}{28\,000} \times 100\% = -3.4\%$

1.8 Component percentage change

Component percentage change is the overall percentage change of an amount due to different components of the amount having different percentage changes.

For example, David's monthly transportation expense is \$400, of which \$160 is spent on travelling by bus and \$240 by MTR. If the bus fare and MTR fare are increased by 5% and 8% respectively, then

New monthly transportation expense $= \$160(1 + 5\%) + \$240(1 + 8\%) = \$427.2$

Percentage change $= \dfrac{427.2 - 400}{400} \times 100\% = 6.8\%$

Demonstration

Section A(1)

1. Peter gets 50 marks and 60 marks in the first and the second mathematics tests respectively.

   (a) Find the percentage increase in his marks.

   (b) If he makes an improvement in the third test with the same percentage, find the mark he gets in the third test.

   (4 marks)

Suggested Solution

(a)   Percentage increase

$= \dfrac{60 - 50}{50} \times 100\%$ 1M

$= \underline{20\%}$ 1A

(b)   The marks he gets in the third test

$= 60(1 + 20\%)$ 1M

$= \underline{72 \text{ marks}}$ 1A

Alternative Method

   The marks he gets in the third test

$= 50(1 + 20\%)^2$ 1M

$= \underline{72 \text{ marks}}$ 1A

> **Guidelines**
> Consider the change from the 2nd to the 3rd test.

> **Guidelines**
> Consider the change from the 1st to the 3rd test. Both stages increased by 20%.

2. The base and the height of a triangle are increased by 15% and decreased by 20% respectively. Find the percentage change in the area.   (4 marks)

Suggested Solution

Let the original base, height and area of the triangle be b, h and A respectively.

The new base $b' = b(1 + 15\%)$

                    $= 1.15\, b$

The new height $h' = h(1 - 20\%)$

                    $= 0.8\, h$

The new area $A' = \dfrac{b'h'}{2}$

                    $= \dfrac{(1.15\, b)(0.8\, h)}{2}$ 1M

                    $= 0.45\, bh$ 1

The original area $A = \dfrac{bh}{2}$

                    $= 0.5\, bh$

The percentage change of the area $= \dfrac{A' - A}{A} \times 100\%$

$= \dfrac{0.45\,bh - 0.5\,bh}{0.5\,bh} \times 100\%$ 1M

$= -8\%$ 1A

Guidelines

The negative percentage change corresponds to a decrease in the area.

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3. Ivy bought a car at $150 000. She sold it to a trading company at a loss percentage of 60%. The company then sold the car at $100 000. Find the profit or loss percentage of the company.   (4 marks)

Suggested Answer

The amount of the car paid by the company

$= \$150\,000(1 - 60\%)$ 1M

$= \$60\,000$ 1

The profit percentage of the company

$= \dfrac{100\,000 - 60\,000}{60\,000} \times 100\%$

    1M

$= 66.7\%$ 1A

Guidelines

The amount paid by the company
= Ivy's selling price
= Cost × (1 – Loss percentage)

Guidelines

∵ $100 000 > $60 000
∴ There is a profit.

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4. A businessman bought ten dozens of watches at the price of $36 000. He set the marked price of each watch at $500, then he sold 40% of the watches at a 10% discount and the rest at a 20% discount. Find his overall profit or loss.   (6 marks)

Suggested Answer

Number of the watches sold at a 10% discount

$= 120 \times 40\%$

$= 48$ 1M

The total selling price of the watches sold at a 10% discount

$= \$500(1 - 10\%) \times 48$

$= \$21\,600$ 1M

Number of the watches sold at a 20% discount

$= 120(1 - 40\%)$

$= 72$ 1M

The total selling price of the watches sold at a 20% discount

$= \$500(1 - 20\%) \times 72$

$= \$28\,800$ 1M

Guidelines

48 watches were sold at a 10% discount.

Practice

* Unless otherwise specified, numerical answers should be exact or correct to 3 significant figures.

* The diagrams are not necessarily drawn to scale.

Section A(1)

1. The electricity cost of a factory in January was $14 000. As the rate on electricity was going to be increase by 5%, the boss of the factory launched an energy saving scheme. As a result, the electricity consumption in February was reduced by 5%. Find the electricity cost of the factory in February.

2. Victor spends $1200 on transportation each month, which corresponds to 15% of his total income. If his salary will be increased by 10% next month, what percentage of his income will be spent on transportation next month? [Hint 1]

3. A lawyer's income is decreased from $y to $\frac{4}{5}y$.

   (a) Find the percentage decrease in his income.

   (b) If his new income is $52 000, find his original income.

4. A metal sphere is heated so that its diameter is increased by 20%. Find the percentage increase in the volume and surface area of the sphere. [Hint 2]

5. A man's weight increased by 4% last month and 6% this month. Find the percentage increase in his weight in these two months.

6. A metal tube is heated in an oven so that its length is increased by 0.2% per minute. Find the percentage increase in the length of the tube after being heated for an hour. [Hint 3]

7. The base and height of a rectangle are increased by 25% and decreased by 10% respectively. Find the percentage change in the area.

8. George's height increased by r% each year at different ages as shown in table 1.1. His height was 110 cm when he was 9.

Age	6 – 11	11 – 14	14 – 18
r% (p.a.)	3%	4%	7%

Table 1.1

   (a) Find his height when he was 7.

   (b) Find his height when he was 12.

   (c) Find his height when he was 18.
   (Give the answers correct to 1 decimal place.)

1 Percentages

Section A (1)

1. The electricity cost in February
    $= \$14\,000(1 + 5\%)(1 - 5\%)$
    $= \underline{\underline{\$13\,965}}$

2. Let his original income and new income be $\$I_1$ and $\$I_2$
    respectively.
    $$I_1 (15\%) = 1200$$
    $$I_1 = \frac{1200}{0.15}$$
    $$I_1 = 8000$$
    $$I_2 = I_1 (1 + 10\%)$$
    $$= 8000 (1 + 10\%)$$
    $$= 8800$$

    The percentage of his income will be spent on
    transportation next month
    $$= \frac{1200}{8800}$$
    $= \underline{\underline{13.6\%}}$ *(correct to 3 significant figures)*

3. (a) Percentage decrease $= \dfrac{y - \dfrac{4}{5}y}{y} \times 100\%$
    $$= \underline{\underline{20\%}}$$

    (b) His new income $= \$52\,000$
    $$\therefore \qquad \frac{4}{5}y = 52\,000$$
    $$y = 52\,000\left(\frac{5}{4}\right)$$
    $$y = 65\,000$$
    His original income $= \underline{\underline{\$65\,000}}$

4. Let the original radius be r.
    The new radius $= r(1 + 20\%)$
    $$= 1.2r$$
    The original volume $= \dfrac{4}{3}\pi r^3$
    The new volume $= \dfrac{4}{3}\pi(1.2r)^3$
    $$= \frac{288}{125}\pi r^3$$
    The percentage increase in the volume
    $$= \frac{\dfrac{288}{125}\pi r^3 - \dfrac{4}{3}\pi r^3}{\dfrac{4}{3}\pi r^3} \times 100\%$$
    $$= \underline{\underline{72.8\%}}$$

The original surface area $= 4\pi r^2$
The new surface area $= 4\pi(1.2r)^2$
$$= 5.76\pi r^2$$
The percentage increase in the surface area
$$= \frac{5.76\pi r^2 - 4\pi r^2}{4\pi r^2} \times 100\%$$
$$= \underline{\underline{44\%}}$$

5. Let his original weight be W.
    His weight after the 4% increase $= W(1 + 4\%)$
    $$= 1.04W$$
    His weight after the 6% increase $= 1.04W(1 + 6\%)$
    $$= 1.1024W$$
    The increase percentage $= \dfrac{1.1024W - W}{W} \times 100\%$
    $$= \underline{\underline{10.24\%}}$$

6. Let the original length be L.
    The new length $= L(1 + 0.2\%)^{60}$
    $$= L(1.002)^{60}$$
    The increase percentage $= \dfrac{L(1.002)^{60} - L}{L} \times 100\%$
    $$= \underline{\underline{12.7\%}}$$

    Reminder

    1 hour = 60 minutes

7. Let the original base, height and area be b, h and A
    respectively.
    The new base $b' = b(1 + 25\%) = 1.25\,b$
    The new height $h' = h(1 - 10\%) = 0.9\,h$
    The new area $A' = \dfrac{b'h'}{2} = \dfrac{(1.25b)(0.9h)}{2} = 0.5625bh$
    The original area $A = \dfrac{bh}{2} = 0.5bh$
    Percentage change of the area
    $$= \frac{A' - A}{A} \times 100\%$$
    $$= \frac{0.5625bh - 0.5bh}{0.5bh} \times 100\%$$
    $$= 12.5\%$$
    The percentage increase in the area is 12.5%.