Exam Paper Format

The HKCE Mathematics Examination, starting from 2006, consists of two papers.

<table>
<tr><th></th><th colspan="3">Paper 1</th><th colspan="2">Paper 2</th></tr>
<tr><td>Types of questions</td><td colspan="3">Conventional questions</td><td colspan="2">Multiple-choice questions</td></tr>
<tr><td>Duration</td><td colspan="3">2 hours</td><td colspan="2">1 hour 30 minutes</td></tr>
<tr><td>Percentage share of the total marks</td><td colspan="3">60%
33 marks [Section A (1)]
33 marks [Section A (2)]
33 marks [Section B]</td><td colspan="2">40%
$\frac{2}{3}$ of the paper mark (Section A)
$\frac{1}{3}$ of the paper mark (Section B)</td></tr>
<tr><td rowspan="2">Details of the papers</td><td rowspan="2">Section A (1)
• 8 to 10 compulsory short elementary questions from the foundation syllabus
• Each question consists of 1 to 2 parts.</td><td rowspan="2">Section A (2)
• 4 to 5 compulsory harder questions from the foundation syllabus
• Each question consists of 2 to 4 parts</td><td rowspan="2">Section B
• 3 out of 4 demanding questions from the whole syllabus
• Each question consists of 3 to 5 parts</td><td>Section A
• 36 compulsory questions from the foundation syllabus</td><td>Section B
• 18 compulsory questions from the whole syllabus</td></tr>
<tr><td colspan="2">• Totally consists of 54 multiple-choice questions
• Four options will be provided in each question.</td></tr>
</table>

Past Exam Questions Distribution (Paper 1)

Topics \ Years	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005
Percentages	12	10a	7	17a	10b	8, 14b	6b	5	3	6, 16a(i), (ii)
Estimation, Rates, Ratios, and Variations	13a	/	12	6	8	13a	11a	10a	10a	5, 10a
Polynomials and Formulas	1, 4	1	5,9	2, 15b	1,6	2,6	4	1, 3	2, 6	1, 3, 10b
Indices, Surds and Logarithms	2	2, 3, 10b	4	1	2	1	1	4	1	2, 16b
Functions and Graphs	/	13	/	7	/	13c	/	/	4	/
Equations	4, 13a, 13b, 13c	8	/	15b	10a	13b, 16a	/	6	7	/
Inequalities and Linear Programming	5, 9b, 9c	4	18	3, 17b(ii)	5, 15	4, 15a, 15b(i)	17, 17b	2, 10b	10b	4
Trigonometry	15,17b, 17c	6, 12a(i)	3, 17	4, 9a, 18	4, 17	9, 16b 17a(ii)	3, 14	9, 14, 15a(ii)	5, 17	14
Sequences	3	15	13	17b(i)	14	12a, 12b(ii)	13b, 13c	7, 15b(ii)	15	7, 16a(iii)
Mensuration	7a, 8, 16a	5c, 7a, 12a(ii) 12b(i), (ii), (iii)	1, 16a	9b, 13	3, 18a, 18b(i), (ii)	3, 9, 12b(i), 16a	2, 6a, 11b, 13a, 15a(i), 15b	13, 15(a)(i)	9, 12b(ii)	9, 12, 13c
Deductive Geometry	10		2, 6, 14	14	13	11	10	8, 15a(iii), 15b(i)	12a, 12b(i)	8, 17a(ii)
Circles	6	9, 16a	6, 14	5, 16a, 16b(iii)	7, 16a, 16b(i)	5, 17b	9, 16a	17a	16a, 16b, 16c(i)	17a(i), 17b(ii)
Coordinate Geometry	9a, 11	16b	8, 15	10, 16b(i), (ii)	9, 16b(ii)	7, 17a(i)	8, 16b, 17a	12, 17b	13, 14a, 14c, 16c(ii)	13a, 13b,17b(i)
Probability	7b	14	11	12	12	15b(ii)	12c	16	8	11, 15c
Statistics	14	11	10	8, 11	11	10	5, 12a, 12b	11	11	15a, 15b

Exam Strategies

A. General Strategies

1. In the examination centre

- Make sure your watch matches with that of the examination centre.
- Listen carefully to the invigilator for any errors and changes in the examination papers.
- Read carefully the instructions on the cover of the answer book or question book.
- Check carefully whether there are any omitted or blank pages in the examination paper or not according to the invigilator's instruction.

2. During the examination

- Attempt the required number of questions. (Please refer to Exam Paper Format on p v.)
- Use proper stationeries.
  - Paper 1: use a pen mainly, but a HB pencil for drawing.
  - Paper 2: use a HB pencil.
- Show your work clearly and neatly.
- Do not be stuck in any one of the questions. Skip it and go on to another one.

3. After finish answering the questions

- Do not be tempted to leave early.
- Check whether there are any questions missed out or not.
- Go back to questions skipped earlier.
- Check whether there are any careless mistakes or not.
- Do not cross out anything before you find enough time to replace it correctly.
- Make sure you write your candidate number on the answer book, supplementary answer sheets and multiple-choice answer sheet.

B. Specific Strategies

1. Paper 1 (2 hours)

- Allocate a reasonable proportion of time to each section and allow 5 minutes for final checking.

Sections	Suggested Time Allocation	Approximate Time per Question
A (1)	30 minutes	3 – 4 minutes
A (2)	40 minutes	8 – 10 minutes
B	45 minutes	15 minutes

- Make your draft in the draft paper provided.
- Show your formulas and steps rather than just writing down the answers. In case you do not get the correct answer, you can get the marks for the correct methods used.

11 Mensuration

Review

11.1 Planes figures

(a) Rectangle

Perimeter = $2(a + b)$

Area = ab

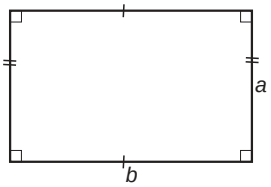

Figure 11.1

(b) Square

Perimeter = $4a$

Area = a^2

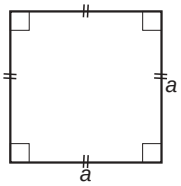

Figure 11.2

(c) Triangle

Perimeter = $a + b + c$

Area = $\frac{1}{2}bh$

NF Area = $\frac{1}{2}ab\sin\theta$

NF Area = $\sqrt{s(s-a)(s-b)(s-c)}$ where $s = \frac{a+b+c}{2}$ (Heron's formula)

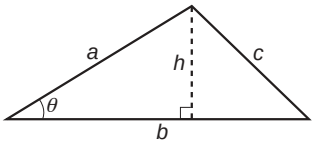

Figure 11.3

(d) Parallelogram

Perimeter = $2(a + b)$

Area = bh

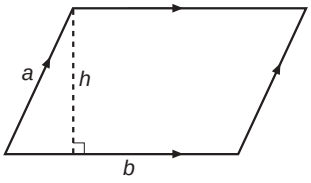

Figure 11.4

4. If $\theta = \phi$, then $AB = BC$ (or $\overset{\frown}{AB} = \overset{\frown}{BC}$). (*equal ∠s, equal chords / arcs*)
   Conversely, if $AB = BC$ (or $\overset{\frown}{AB} = \overset{\frown}{BC}$), then $\theta = \phi$. (*equal chords / arcs, equal ∠s*)

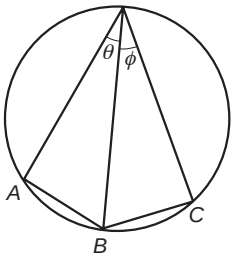

Figure 13.14

5. $\overset{\frown}{AB} : \overset{\frown}{CD} = \theta : \phi$ (*arcs prop. to ∠s at* $\odot^{ce}$)

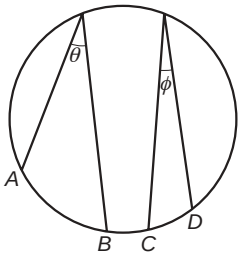

Figure 13.15

Reminder

$AB : CD \neq \theta : \phi$

For example, in Figure 13.16, $\overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{AC} = 1 : 2 : 1$.
Find $\angle BAC$.
Since $\overset{\frown}{AB} : \overset{\frown}{BC} : \overset{\frown}{AC} = 1 : 2 : 1$,
$\therefore\ \angle BCA : \angle BAC : \angle ABC = 1 : 2 : 1$
(*arcs prop. to ∠s at* $\odot^{ce}$)
Since the sum of angles of a triangle is 180°,

$$\therefore\ \angle BAC = 180° \times \frac{2}{1 + 2 + 1}$$
$$= 90°$$

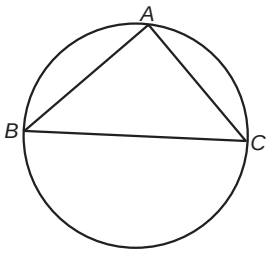

Figure 13.16

Reminder

If $X : Y : Z = a : b : c$ and $X + Y + Z = P$, then

$$X = \frac{a}{a + b + c} \times P\ ;$$
$$Y = \frac{b}{a + b + c} \times P\ ;$$
$$Z = \frac{c}{a + b + c} \times P\ .$$

13.4 Cyclic quadrilaterals

If $ABCD$ is a cyclic quadrilateral, then
(i) $\angle BAD + \angle BCD = 180°$ and $\angle ADC + \angle ABC = 180°$. (*opp. ∠s, cyclic quad.*)
(ii) $\angle DCE = \angle BAD$. (*ext. ∠, cyclic quad.*)

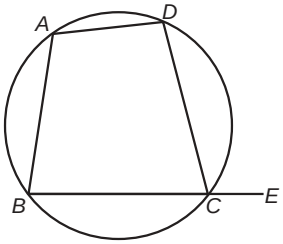

Figure 13.17

Demonstration

Section A(1)

1. Consider the points $A(5, -1)$ and $B(-2, 6)$. Find

   (a) the distance between A and B;

   (b) the mid-point of A and B.

   (Express the answer in surd form if necessary.) (3 marks)

Suggested Solution

(a) $AB = \sqrt{(5-(-2))^2 + (-1-6)^2}$ 1M

$= \sqrt{7^2 + (-7)^2}$

$= \sqrt{98}$

$= \underline{\underline{7\sqrt{2} \text{ units}}}$ 1A

(b) The coordinates of the mid-point of A and $B = \left(\frac{5+(-2)}{2}, \frac{-1+6}{2}\right)$

$= \underline{\underline{\left(\frac{3}{2}, \frac{5}{2}\right)}}$ 1A

Guidelines

Distance

$= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Guidelines

Mid-point

$= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

N F 2. If a point $P(2, y)$ divides the line segment AB in the ratio $1: s$ internally where $A = (3, 5)$ and $B = (-1, -4)$, find the values of s and y.

(3 marks)

Suggested Solution

$P(2, y) = \left(\frac{1(-1) + s(3)}{1+s}, \frac{1(-4) + s(5)}{1+s}\right)$ 1M

Equating the x and y coordinates, we have

$2 = \frac{-1+3s}{1+s}$ .....(1)

and $y = \frac{-4+5s}{1+s}$ .....(2)

From (1), $2 + 2s = -1 + 3s$ 1A

$-s = -3$

$s = \underline{\underline{3}}$

Substituting $s = 3$ into (2), $y = \frac{-4+5(3)}{1+(3)} = \underline{\underline{\frac{11}{4}}}$ 1A

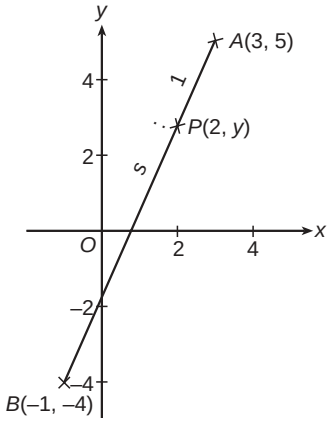

Figure 14.18

$$\frac{781}{1024} = 1 - \left(\frac{3}{4}\right)^n$$

$$-\frac{243}{1024} = -\left(\frac{3}{4}\right)^n$$

$$\left(\frac{3}{4}\right)^5 = \left(\frac{3}{4}\right)^n \quad \text{1M}$$

$$n = 5 \quad \text{1A}$$

Guidelines

Express $\frac{243}{1024}$ as a power of $\frac{3}{4}$.

Guidelines

If $a^m = a^n$, then $m = n$.

Section A(2)

NF 7. (a) Find the smallest and the largest multiples of 6 between 100 and 599 inclusive.

(b) How many multiples of 6 are there between 100 and 599 inclusive?

(c) Find the sum of all multiples of 6 from 100 to 599 inclusive.

(7 marks)

Suggested Solution

(a) The smallest and the largest multiples of 6 between 100 and 599 inclusive are 102 and 594 respectively. 1A + 1A

(b) The multiplies of 6 between 100 and 599 inclusive form an arithmetic sequence with first term $a = 102$, common difference $d = 6$ and last term = 594. 1M

$T(n) = a + (n - 1)d$

$594 = 102 + (n - 1)(6)$ 1M

$594 = 6n + 96$

$6n = 498$

$n = 83$

$\therefore$ There are 83 multiples of 6 between 100 and 599 inclusive. 1A

(c) $S(83) = \frac{83}{2}(102 + 594)$ 1M

$= 28\,884$ 1A

$\therefore$ The sum of all multiples of 6 from 100 to 599 inclusive is 28 884.

Guidelines

The first term is 102, the last term is 594 and there are 83 terms between 100 and 599 inclusive.

NF 8. The number of applications for *ABC* credit card is 20 500 at the end of 2004. If the number of applications is increased by 5% in the subsequent years, find

(a) the number of applications for *ABC* credit card at the end of 2005;

Practice

* Unless otherwise specified, numerical answers should be either exact or correct to 3 significant figures.
* The diagrams are not necessarily drawn to scale.

Section A(1)

1. Find the mean, the mode and the median of 4, 9, 13, 19, 19, 21 and 27.

2. Consider a set of data: 0.9, 1.2, 1.5, 0.5, 0.3 and 1.5. Find the mean, the mode and the median of the data.

NF 3. If 5, y and 25 form an arithmetic sequence, find

   (a) the value of y;

   (b) the mean.

NF 4. If 4.5, a and 2 form a geometric sequence, find

   (a) the values of a;

   (b) the common ratios;

   (c) the means.

5. Find the range, the inter-quartile range and the standard deviation of 4, 5, 6, 7, 9 and 11. (Working steps are not required for finding the standard deviation.)

6. Find the range, the inter-quartile range and the standard deviation of 3.1, 8.5, 10.3, 13.5, 17.9, 21.4, 25.2 and 29.8. (Working steps are not required for finding the standard deviation.)

NF 7. The mean of 5 numbers is 25. If 6 and 7 are added to this set of data, find the new mean. Hint 1

8. Consider a set of data arranged in ascending order: a, 5, 7, b, 13 and 18. If the median and the mean of this set of data are 9 and 9.5 respectively, find the values of a and b.

NF 9. Consider a set of data: 2, 8, 12, 16, 20, x and y. If the mean and the standard deviation of this set of data are 11 and $\sqrt{\frac{202}{7}}$ respectively, find the value of xy. Hint 2

10. Find the mean and the inter-quartile range of x, x, $x+2$, $x-3$, $x+5$ and $x-7$.

15 Probability

Section A (1)

1. $P(\text{prize})$
$= P(2 \text{ or } 4)$
$= \frac{2}{6}$
$= \underline{\underline{\frac{1}{3}}}$

2. (a) $P(\text{tail})$
$= \frac{81}{300}$
$= \underline{\underline{\underline{\frac{27}{100}}}}$

(b) $P(\text{head})$
$= \frac{219}{300}$
$= \underline{\underline{\underline{\frac{73}{100}}}}$

3. (a) $P(5) = \frac{32}{400}$
$= \underline{\underline{\underline{\frac{2}{25}}}}$

(b) $P(\text{odd}) = \frac{70 + 56 + 32}{400}$
$= \underline{\underline{\underline{\frac{79}{200}}}}$

(c) $P(\text{less than 3}) = \frac{70 + 53}{400}$
$= \underline{\underline{\underline{\frac{123}{400}}}}$

4. The 24 possible outcomes are listed below:

1st \ 2nd	1	2	3	4
1	0	1	2	3
2	1	0	1	2
3	2	1	0	1
4	3	2	1	0
5	4	3	2	1
6	5	4	3	2

(a) $P(0) = \frac{4}{24}$
$= \underline{\underline{\frac{1}{6}}}$

(b) $P(2) = \frac{6}{24}$
$= \underline{\underline{\frac{1}{4}}}$

(c) $P(6) = \underline{\underline{0}}$

(d) $P(\text{greater than 3}) = \frac{3}{24}$
$= \underline{\underline{\frac{1}{8}}}$

5. (a) $P(\text{even}) = \underline{\underline{\frac{3}{7}}}$

(b) (i) The 42 possible outcomes are listed below:

1st \ 2nd	1	2	3	4	5	6	7
1	×	2	3	4	5	6	7
2	2	×	6	8	10	12	14
3	3	6	×	12	15	18	21
4	4	8	12	×	20	24	28
5	5	10	15	20	×	30	35
6	6	12	18	24	30	×	42
7	7	14	21	28	35	42	×

(ii) $P(\text{reward}) = \frac{30}{42}$
$= \underline{\underline{\frac{5}{7}}}$

N/F Alternative Method
$P(\text{reward})$
$= P(\text{first card is even})\ P(\text{second card is odd}) +$
$P(\text{first card is odd})\ P(\text{second card is even}) +$
$P(\text{both cards are even})$
$= \left(\frac{3}{7} \times \frac{4}{6}\right) + \left(\frac{4}{7} \times \frac{3}{6}\right) + \left(\frac{3}{7} \times \frac{2}{6}\right)$
$= \underline{\underline{\frac{5}{7}}}$