# Linear Inequalities in One Unknown



# 1. Notations

For real numbers *a* and *b*, a > b means that *a* is greater than *b*;

a < b means that a is smaller than b;

 $a \ge b$  means that a is greater than or equal to b;

 $a \le b$  means that *a* is smaller than or equal to *b*;

 $a \neq b$  means that a is not equal to b.

# 2. Properties of Inequalities

#### (a) Transitive Property

If a > b and b > c, then a > c.

#### (b) Additive Property

If a > b and c is a real number, then a + c > b + c.

#### (c) Multiplicative Properties

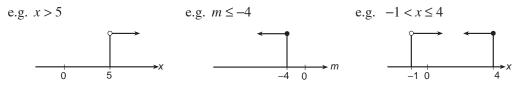
- (i) If a > b and c > 0, then ac > bc.
- (ii) If a > b and c < 0, then bc > ac.
- (iii) If  $a \neq 0$ , then  $a^2 > 0$ .

#### (d) Reciprocal Properties

- (i) If a > b > 0, then  $\frac{1}{b} > \frac{1}{a}$ .
- (ii) If a > b and **both** a and b are negative, then  $\frac{1}{b} > \frac{1}{a}$ .

e.g. 
$$4 > 3 > 0$$
  
 $\therefore \frac{1}{3} > \frac{1}{4}$ 
e.g.  $-2 > -5$   
 $\therefore \frac{1}{-5} > \frac{1}{-2}$ 

# 3. Representation on a Number Line



# 4. Solving Linear Inequalities in One Unknown

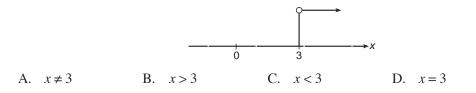
e.g. Solve  $5 - x < \frac{4x - 5}{2}$  and represent the solution on a number line.

Solution: 
$$5 - x < \frac{4x - 5}{2}$$
$$2(5 - x) < 2\left(\frac{4x - 5}{2}\right)$$
$$10 - 2x < 4x - 5$$
$$10 + 5 < 4x + 2x$$
$$15 < 6x$$
$$x > \frac{5}{2}$$

### Level 1

For questions 1 - 4, choose the most suitable answer from the four choices.

- 1. If a > 0, which of the following inequalities must be correct?
  - A. 3a + 5 > 5aB. 3a + 5 > 5C. -a > 0D. a 10 > 10
- 2. What does the following graph represent?

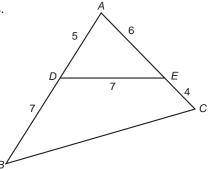


- 3. If a > b > 0 > c, which of the following is incorrect?
  - A. a+b > a+c B. ab > ac
  - C. a b > c b D. ac > b(0)



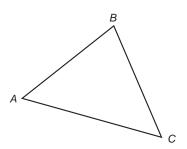
There are mistakes in the solution below. Correct the mistakes.

Prove that  $\triangle ABC \sim \triangle AED$ . Solution:  $\angle EAD = \angle BAC \ (common)$  $\angle AED = \angle ACB$  $\angle ADE = \angle ABC$  $\therefore \ \triangle ABC \sim \triangle AED \ (equiangular)$ 





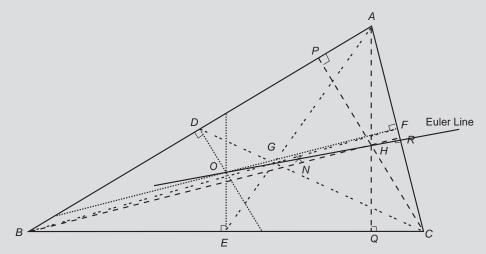
Draw a circle passing through the points *A*, *B* and *C* given below. [Hint: Draw perpendicular bisectors of *AB*, *BC* and *AC*.]





#### **Euler Line**

The orthocentre H, the centroid G and the circumcentre O of any triangle all lie on a straight line, known as the Euler Line. This is illustrated in the figure below.



In the figure, we have GH = 2OG.

If you construct a circle centred at the mid-point *N* of *OH* with radius *NP*, what do you find?

You may visit the following website to see the Euler line of triangles in different shapes:

http://www.cut-the-knot.org/triangle/EulerLine.shtml

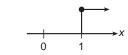
altitude	頂垂線/高	angle bisector	角平分線
bisect	平分	centroid	形心
circumcentre	外心	congruent	全等
deductive geometry	演繹幾何	diagonal	對角線
equiangular	等角	in-centre	內心
intercept theorem	截線定理	median	中線
mid-point theorem	中點定理	orthocentre	垂心
parallelogram	平行四邊形	perpendicular bisector	垂直平分線
proportional	成比例	quadrilateral	四邊形
rhombus	菱形	similar	相似

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Choose the most suitable answer from the four choices.

1. Which of the following inequalities is represented by the diagram below?



- A.  $2x \ge -2$
- B.  $2x 1 \ge 1$
- C.  $-2x \ge -2$
- D.  $3x 2 \le 2x 1$
- 2. If a > b and c < 0, which of the following must be true?
  - ac > bcI.
  - II. a + c > b + c
  - III. -ac > -bc
  - A. II only B. III only
  - C. I and II only D. II and III only
- 3.  $1 \text{ km}^2 =$ 
  - A.  $1 \times 10^{10} \text{ cm}^2$  B.  $1000 \text{ cm}^2$ C.  $1 \times 10^8 \text{ cm}^2$  D.  $1 \times 10^8 \text{ m}^2$
- Which of the following is CORRECT? 4.
  - A.  $(-x)^3 \times (x)^{-5} = x^{-2}$ B.  $(2x^3)^2 = 4x^9$ C.  $(-2^2 x y^{-1})^3 = \frac{64 x^3}{y^3}$ D.  $(-x)^{-1} = \frac{-1}{x}$
- 5. X = A + B + C, A is increased from 400 by 40%, B is increased from 350 by 35%, and C is increased from 250 by 25%. What is the percentage change in X?

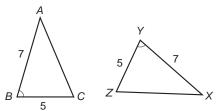
- A. Increased by 100%
- B. Increased by 35%
- C. Increased by 34.5%
- D. Increased by 25%
- x is increased by 10 %, then decreased by 6. 20% and then increased by 10%. What is the percentage change in x?
  - A. Increased by 3.2%
  - B. Unchanged
  - C. Increased by 0.83%
  - D. Decreased by 3.2%
- 7. Which of the following is a rational number?

А.	2π	В.	$\sqrt{24}$
C.	$\sqrt{1+81}$	D.	3.1416

8.  $(\sqrt{3} - 1)(1 + \sqrt{3}) =$ A. 2 C.  $\sqrt{3} - 1$  D.  $2 - \sqrt{3}$ 



9. Which of the following is correct?



B.  $2\sqrt{3}$ 

- A.  $\triangle ABC \cong \triangle XYZ (SAS)$
- B.  $\triangle ABC \cong \triangle XYZ (SSS)$
- C.  $\triangle ABC \cong \triangle XYZ (AAS)$
- D.  $\triangle ABC \sim \triangle XYZ$  (ratio of 2 sides, inc.  $\angle$ )



1. When a number is increased by x% ( $x \neq 0$ ), and then decreased by x%, explain why the result is not equal to the original number.

2. The following shows the tax rate of a country in 2 consecutive financial years.

Financial year	Tax rate	
2006 - 2007	4%	
2007 - 2008	5%	

Mrs. Chan owns a flat and she had to pay quarterly rates of \$2560 in 2006 – 2007. Since the tax rate in 2007 – 2008 was changed from 4% to 5%, Mrs. Chan suggested that the new quarterly rates would also be increased by (5% - 4%) = 1%. Do you agree with her? Explain your answer briefly.