

Chapter 2

Factorization of Polynomials

Junior Secondary

課程焦點

Factorize polynomials:

- by taking out common factors and grouping terms
- by cross-method
- by using identities including difference of two squares and perfect square

NF ▪ by using identities including difference and sum of two cubes

歷屆試題分析

以下展示近年具代表性的公開試考題的題型及難度變化。

Paper 1

(a) 部利用以下方法作因式分解 (factorization)

- 提取公因式 (taking out common factors) [CE 2009]
- 十字相乘法 (cross-method) [DSE 2014]
- 恒等式 (identity) [DSE 2013]

參考 CE 2009 Q3

- (a) Factorize $x^2y - xy^2$.
(b) Hence, factorize $x^2y - xy^2 + 6x - 6y$.

參考 DSE 2014 Q2

- (a) Factorize $x^2 + 4x - 5$.
(b) Hence, factorize $xy^2 - y^2 + x^2 + 4x - 5$.

參考 DSE 2013 Q3

- (a) Factorize $x^2 - 4y^2$.
(b) Hence, factorize $x^2 - 4y^2 + 3x - 6y$.

拆題示例 4

(b) 部利用 (a) 部的結果作因式分解。

參考 CE 2010 Q3

- (a) Factorize $a^2 + 2a + 1$.
(b) Hence, factorize $a^2 + 2a + 1 - b^2$.

拆題示例 5

(b) 部利用 (a) 部的結果及恒等式作因式分解。

Paper 2

參考 CE 2010 Q4

Factorize $r^2 + st - rs - rt$.

重排各項然後作因式分解。

參考 DSE 2013 Q3

Factorize $ax - bx - ay + by + 2a - 2b$.

6 項的多項式 (polynomial) 的因式分解，近年只考過一次。

拆題示例 3

參考 DSE 2014 Q2

Factorize $x^2 - y^2 - 2x + 2y$.

利用恒等式作因式分解，但多項式較 Paper 1 簡單，且不需要重排各項。

參考 Sample Paper Q3

Factorize $a^2 - b^2 + 6b - 9$.

兩次利用恒等式作因式分解。

淺

深

Level 2



要點一覽及評分準則

拆題示例 3

DSE

Level

2

Factorize $ax - bx - ay + by + 2a - 2b$.

(2 marks)

攻略

Polynomial with 6 terms

用 grouping terms :

- 以兩項為一組，或
- 以三項為一組。

Polynomial with 4 terms

- 簡單的題型只需用 grouping terms :
 - 不涉及調項 (見題 1 - 4)
 - 涉及調項 (見題 5 - 6)
- 較深的題型 (見示例 4 及示例 5) 需混合運用
 - Taking out common factors
 - Cross-method
 - Identity

Sol. (參考 DSE 2013 Paper 2 Q3)

$$ax - bx - ay + by + 2a - 2b$$

$$= (ax - bx) - (ay - by) + (2a - 2b) \quad \leftarrow \text{注意正負號。}$$

$$= x(a - b) - y(a - b) + 2(a - b)$$

1M

$$= \underline{(a - b)(x - y + 2)}$$

1A

Alternative Sol.

$$ax - bx - ay + by + 2a - 2b$$

$$= (ax - ay + 2a) - (bx - by + 2b) \quad \leftarrow \text{注意正負號。}$$

$$= a(x - y + 2) - b(x - y + 2)$$

1M

$$= \underline{(a - b)(x - y + 2)}$$

1A

即時考驗

- Factorize $3xw + xz + 3yw + yz$. CE (2 marks)
- Factorize $ax - ay + bx - by$. CE (2 marks)
- Factorize $3x + 6y - ax - 2ay$. CE (2 marks)
- Factorize $b - ab - a + a^2$. CE (2 marks)
- Factorize $xy + z - y - xz$. (2 marks)
- Factorize $r^2 + st - rs - rt$. CE (2 marks)
- Factorize $ax + bx + cx - ay - by - cy$. DSE (2 marks)

進階題

- Factorize $2a + 2b - ac + ay - bc + by$. (2 marks)

拆題示例 4

DSE

Level

2



- (a) Factorize $x^2 - 4y^2$.
 (b) Hence, factorize $x^2 - 4y^2 + 3x - 6y$.

(3 marks)

攻略

Polynomial in (a)

用以下其中一種方法：

- Taking out common factors
- 利用 difference of two squares / perfect square
- Cross-method

Polynomial in (b)

步驟 1

把它分成兩組，其中一組必為 (a) 部的 polynomial。

步驟 2

代入 (a) 部的結果，再 taking out common factors。

注意：Paper 1 [利用 (a) 部結果解答 (b) 部]

Paper 2 [不拆 (a)、(b) 部，一題過，較深]，
 如題 13 - 14。

Sol. (參考 DSE 2013 Paper 1 Q3)

$$\begin{aligned} \text{(a)} \quad & x^2 - 4y^2 \\ &= x^2 - (2y)^2 \quad \leftarrow \text{Difference of two squares} \\ &= \underline{(x - 2y)(x + 2y)} \end{aligned} \quad 1A$$

$$\begin{aligned} \text{(b)} \quad & \text{By using the result of (a),} \\ & \underline{x^2 - 4y^2} + 3x - 6y \\ &= (x^2 - 4y^2) + 3x - 6y \\ &= (x - 2y)(x + 2y) + 3x - 6y \quad 1M \\ &= (x - 2y)(x + 2y) + 3(x - 2y) \\ &= \underline{(x - 2y)(x + 2y + 3)} \quad 1A \end{aligned}$$

即時考驗

9. (a) Factorize $x^2y - xy^2$.
 (b) Hence, factorize $x^2y - xy^2 + 6x - 6y$. (3 marks)

10. (a) Factorize $x^2 + 4x - 5$.
 (b) Hence, factorize $xy^2 - y^2 + x^2 + 4x - 5$. (3 marks)

11. (a) Factorize $25a^2 + 20ab + 4b^2$.
 (b) Hence, factorize $25a^2 + 20ab + 4b^2 - 15a - 6b$. (3 marks)

12. (a) Factorize $9a^2 - 4b^2$.
 (b) Hence, factorize $9a^2 - 4b^2 - 3a - 2b$. (3 marks)

13. Factorize $x^2 - y^2 - 2x + 2y$. (3 marks)

14. Factorize $16x^2 - 9y^2 - 20x - 15y$. (3 marks)

- NF 15. (a) Factorize $a^3 - b^3$.
 (b) Hence, factorize $a^3 - b^3 - 2a + 2b$. (3 marks)



要點一覽及評分準則

拆題示例 80

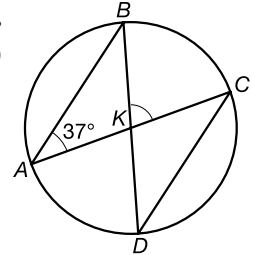
DSE

Level

4

In the figure, $ABCD$ is a circle. AC intersects BD at K . If $AB \parallel DC$ and $\angle BAC = 37^\circ$, find $\angle BKC$.

(3 marks)



攻略

- 此類題型主要利用平行線上的角及圓內的角的性質。
- 注意兩個三角形 (示例中 $\triangle ABK$ 及 $\triangle CDK$) 組成的蝴蝶形, 這提示:
 - a. 常與「angles in the same segment」有關。
 - b. 當題目包含平行線時, 常與「alternate angles」有關。
- 注意題設沒有指明 K 點是圓心, 因此不能利用「angle at centre twice angle at circumference」求 $\angle BKC$ (雖然可得出相同結果, 原因請以 QR-code 查閱本章之補充資料)。

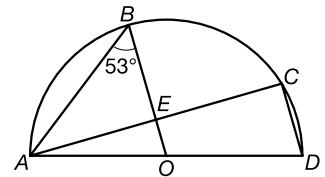
Sol. (參考 DSE 2013 Paper 2 Q19)

$$\begin{aligned} \angle KCD &= \angle BAC \quad (\text{alt } \angle s, AB \parallel DC) \\ &= 37^\circ && 1A \\ \angle CDB &= \angle BAC \quad (\angle s \text{ in the same segment}) \\ &= 37^\circ && 1A \\ \angle BKC &= \angle KCD + \angle CDB \quad (\text{ext. } \angle \text{ of } \triangle) \\ &= 37^\circ + 37^\circ \\ &= \underline{74^\circ} && 1A \end{aligned}$$

即時考驗

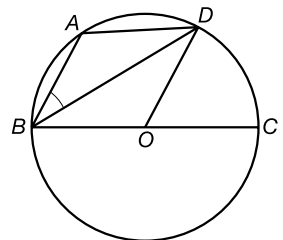
1. In the figure, O is the centre of the semicircle $ABCD$. AEC and BEO are straight lines. If $BO \parallel CD$ and $\angle OBA = 53^\circ$, find
 - (a) $\angle AOB$,
 - (b) $\angle CAD$.

DSE (5 marks)



2. In the figure, O is the centre of the circle and BOC is a straight line. $BA \parallel OD$ and $\angle ABC = 62^\circ$. Find $\angle DBA$.

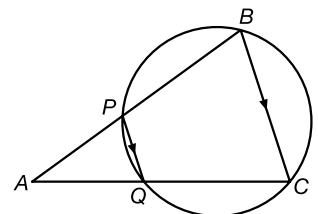
CE (4 marks)



3. In the figure, APB and AQC are straight lines. $PQ \parallel BC$ and $\angle ACB = 72^\circ$.

- (a) Find $\angle AQP$ and $\angle ABC$.
- (b) Find $\angle PAQ$.

CE (4 marks)



拆題示例 110

DSE

Level

4

There are 5 boys and 10 girls in a class. 4 students are selected to form a group. If the group consists of at least one boy, how many different groups can be formed? (3 marks)

攻略

- 此示例為示例 109 的變化題型 (較深)，注意重點字眼如「at least」及「at most」。

較快做法

有以上重點字眼的題目可用 complement 的概念解題：

$$T = C + C'$$

T ：沒有任何限制下的 combination 總數目

C ：題設條件下的 combination 數目 (答案)

C' ：與題設條件相反下的 combination 數目

一般做法

直接把所有可能情況的 combination 數目相加求解，但要考慮的情況或會較多。

- 此類題型常涉及 probability。(見 Chapter 33 示例 114)。

Sol. (參考 DSE 2012 Paper 2 Q43)

$$\begin{aligned} &\text{Number of groups formed with no restrictions} \\ &= C_4^{15} \qquad \qquad \qquad 1M \end{aligned}$$

$$= 1365 \qquad \leftarrow T$$

$$\begin{aligned} &\text{Number of groups formed with no boys} \\ &= C_0^5 \times C_4^{10} \qquad \qquad \qquad 1M \end{aligned}$$

$$= 210 \qquad \leftarrow C'$$

$$\begin{aligned} &\text{Number of groups formed with at least one boy} \\ &= 1365 - 210 \qquad \leftarrow T - C' \end{aligned}$$

$$= \underline{1155} \qquad \leftarrow C \qquad \qquad \qquad 1A$$

Alternative Sol.

$$\begin{aligned} &\text{Number of groups formed with at least one boy} \\ &= C_4^5 \times C_0^{10} + C_3^5 \times C_1^{10} + C_2^5 \times C_2^{10} + C_1^5 \times C_3^{10} \quad 1M + 1A \end{aligned}$$

$$= \underline{1155} \qquad \qquad \qquad 1A$$

即時考驗

- If a group of 4 people are selected from 6 pairs of man and woman, find the number of ways that there are at most 2 women in the group. DSE (3 marks)
- There are seven cards labelled 1, 2, 3, 4, 5, 6 and 8 respectively. Three cards are drawn and arranged in a row. How many different arrangements can have at least one odd number? (3 marks)

進階題

- 12 different coins are tossed. Find the number of possible outcomes that at least 2 'head's are obtained? (3 marks)

自我評估表

利用二維碼 (QR-code) 查閱評分準則，並利用以下評分表，評一評自己的表現：

即時考驗得分

1.	2.	3.	4.	5.	6.	7.	8.	9.

總分：_____ / 23

評分表

完成點數	表現
<input type="checkbox"/> 18 - 23	 優異
<input type="checkbox"/> 12 - 17	 一般
<input type="checkbox"/> 0 - 11	 加油